

## WAMC Lab Template

Math Concept(s): Interpreting Functions, FI-F8b

Source / Text: Financial Algebra, 2<sup>nd</sup> Edition, Gerver/Srgoi, Section 3-7

Developed by: Kari Toms E-Mail: ktoms@eagles.edu

Date: Summer In-service 2014

### Attach the following documents:

Lab Instructions

Student Handout(s)

Rubric and/or Assessment Tool

### Short Description:

### Lab Plan

Lab Title: Paper Money Grows

Prerequisite skills: Future Value Formula:  $FV = P(1 + r/n)^{nt}$

Lab objective: To forecast and visibly demonstrate the variable growth afforded by various levels of interest compounded into future values.

1. Estimate future values
2. Calculate future values over multiple periods
3. Observe impacts of compounding influences

### Standards:

CCSS-M:

- F-IF8b, Interpreting Functions

Standards for Mathematical Practice:

- MP.4: Model with mathematics

State Standards addressed (2008 Washington State Mathematics Standards):

- A1.1.A Select and justify functions and equations to model and solve problems

### Leadership/21st Century Skills:

<u>21st Century Interdisciplinary themes</u> (Check those that apply to the above activity.)			
<input type="checkbox"/> Global Awareness	<input checked="" type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy	<input type="checkbox"/> Civic Literacy	
<input type="checkbox"/> Health/Safety Literacy	<input type="checkbox"/> Environmental Literacy		
<u>21st Century Skills</u> (Check those that students will demonstrate in the above activity.)			
<b>LEARNING AND INNOVATION</b>	<b>INFORMATION, MEDIA &amp; TECHNOLOGY SKILLS</b>	<b>LIFE &amp; CAREER SKILLS</b>	<b>Productivity and Accountability</b>
<u>Creativity and Innovation</u>	<u>Information Literacy</u>	<u>Flexibility and Adaptability</u>	<input type="checkbox"/> Manage Projects
<input type="checkbox"/> Think Creatively	<input type="checkbox"/> Access and Evaluate Information	<input type="checkbox"/> Adapt to Change	<input type="checkbox"/> Produce Results
<input type="checkbox"/> Work Creatively with Others	<input type="checkbox"/> Use and manage Information	<input type="checkbox"/> Be Flexible	<input type="checkbox"/> Guide and Lead
<input type="checkbox"/> Implement Innovations	<u>Media Literacy</u>	<u>Initiative and Self-Direction</u>	<input type="checkbox"/> Others
<u>Critical Thinking and Problem Solving</u>	<input type="checkbox"/> Analyze Media	<input type="checkbox"/> Manage Goals and Time	<input type="checkbox"/> Be Responsible to Others
<input type="checkbox"/> Reason Effectively	<input type="checkbox"/> Create Media Products	<input type="checkbox"/> Work Independently	<input checked="" type="checkbox"/> Be Responsible to Others
<input type="checkbox"/> Use Systems Thinking	<u>Information, Communications and Technology (ICT Literacy)</u>	<input type="checkbox"/> Be Self-Directed Learners	
<input checked="" type="checkbox"/> Make Judgments and Decisions	<input type="checkbox"/> Apply Technology Effectively	<u>Social and Cross-Cultural</u>	
<input checked="" type="checkbox"/> Solve Problems		<input type="checkbox"/> Interact Effectively with Others	
<u>Communication and Collaboration</u>		<input checked="" type="checkbox"/> Work Effectively in Diverse Teams	
<input type="checkbox"/> Communicate Clearly			
<input checked="" type="checkbox"/> Collaborate with Others			

## **Teacher Preparation: (What materials and set-up are required for this lab?)**

### Materials

- Scratch Paper
- Scientific Calculator
- Lab Direction Sheets
- Glue Sticks, 1-2 per group
- Up to 100 cut paper segments of equal size
- One future investment scenario card for each group

### Set-Up Required:

- Group Tables, with up to 4 people per group
- A calculator per group
- At least one glue stick per group

### **Lab Organization Strategies:**

#### Grouping/Leadership/Presentation Opportunities:

- Students will have to take turns and check each others' work

#### Cooperative Learning:

- Forecasting and communicating that forecast with

#### Expectations:

- Students will be able to estimate a future value for a compounded amount, represent it visually, and prove their estimate mathematically.

#### Timeline:

- 1 class period (55 minutes)

### **Post Lab Follow-Up/conclusions:**

#### Discuss real world application of learning from lab

- Estimate amount of earnings with compounding
- Savings and Investment Accounts

#### Career Applications

- Calculating investments

#### Optional or Extension Activities

- MS Excel Scatter Plots or trends can be added to forecast additional periods for extended activities
- Rates and cash flows can change each period

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# Paper Money Grows

## Objectives:

1. Estimate future values
2. Calculate future values over multiple periods
3. Verify estimates using formula

## Procedure:

1. Divide into groups of four people. Move and gather around a table with your group.
2. One person should draw a scenario card from the teacher. Place it face down on your table.
3. When instructed, turn over your card and as a group, estimate the future value of the investment account on your card after 5 years, if you begin with \$1000.
4. One person should write your group name and estimate on the whiteboard in front of the classroom.
5. Another person should collect a scientific calculator, glue stick, a stack paper rectangles and scratch paper for your group.
6. Each paper rectangle represents \$100. As a group, take a few minutes to make a paper chain representing \$1000 (10 chain links) to visually represent the initial investment. Each paper rectangle should be glued at one end and glued to the other end to make a link. The second link should be linked through the center of the first link before gluing to make 10 links.
7. For team member 1, have them use the future value formula to calculate the value of your investment account after one year. Record the value on the board next to your group estimate.
8. As a team, make additional paper chain links to extend your original 10 link chain, rounding your value at the end of year one to the nearest \$100 dollars. Write on the last link in this segment, 'Year 1'.
9. For the next team member, have them calculate the value for the end of year 2. Write the value on the board, then have your team assemble paper chains to represent the 100s of dollars (rounded to the nearest hundred dollars) and attach it to your chain.
10. Repeat step 10 for years 3, 4, and 5.
11. One person should calculate the difference between your estimated value and your calculated value. Write this value on the board for your group.
12. When instructed, all groups will take their paper chains out to the hallway to make a physical bar graph, starting at the west end of the hall.
13. Observe the lengths of the paper chain's growth. Consider the factors contributing to length or lack thereof, in other groups.
14. When instructed, all groups will clean-up their area and return materials to the supply area.

## POST-LAB QUESTIONS

1. Which group had the longest chain? How many 100s of dollars was their investment 'worth'?
2. How did your group estimate their value?
3. Were there any possible sources of error in calculating the future value of your investment?
4. Which group had the shortest chain? Why do you think that their investment grew the least?
5. What do you think has the greatest impact on compounding?

RUBRIC AREA	POINTS
Observed Teamwork	4
Forecast is made on board	2
Values are Calculated for each year	5
Chain represents compound growth accurately is made with correct links, stays linked)	5
Post-lab questions are answered thoughtfully	5
Differential between estimate and proven value is shown	2
Work-area is clean after lab	4

# Washington

## Group #1

Your investment is in a CD, with quarterly compounded interest. You will earn 8.25% yearly.

## Group #2

Your investment is in a bond account. You will earn 9% interest, compounded annually.

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## Group #3

Your investment is in a money market account. Your interest will be compounded semi-annually at a rate of 5.5%.

## Group #4

Your investment is in a high-yield savings account. Your interest is compounded monthly at a nominal rate of 2.15%

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# Washington

## Group #5

Your investment is in an interest-bearing checking account. You will earn 3.25% nominally, with monthly compounded interest.

## Group #6

Your investment is in a CD. It earns 3% interest, which is compounded annually.

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## Group #7

You have invested in bond fund. You will earn 6.25% interest annually, which is compounded quarterly.

## Group #8

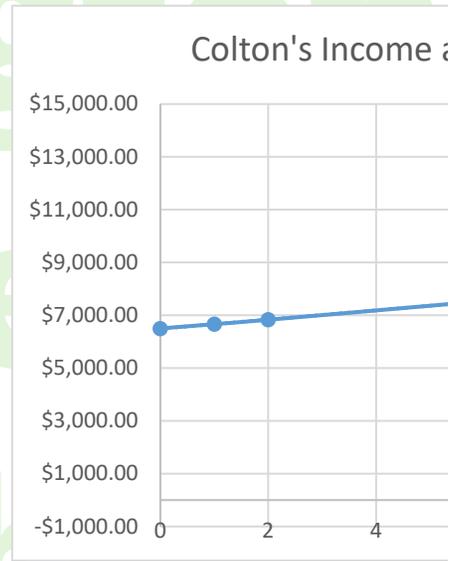
Your investment is in a mutual fund. It is expected to earn 7.875% yearly, with monthly compounded interest.

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Period	Account Balance	Deposits	Subtotal	Interest Added
	\$0.00	0	\$0.00	\$0.00
1	\$0.00	25	\$25.00	\$0.00
2	\$25.00	25	\$50.00	\$0.00
3	\$50.00	25	\$75.00	\$2.25 QTR1
4	\$77.25	25	\$102.25	\$0.00
5	\$102.25	25	\$127.25	\$0.00
6	\$127.25	25	\$152.25	\$4.57 QTR2
7	\$156.82	25	\$181.82	\$0.00
8	\$181.82	25	\$206.82	\$0.00
9	\$206.82	25	\$231.82	\$6.95 QTR3
10	\$238.77	25	\$263.77	\$0.00
11	\$263.77	25	\$288.77	\$0.00
12	\$288.77	25	\$313.77	\$9.41 Q4
13	\$323.19	25	\$348.19	\$0.00
14	\$348.19	25	\$373.19	\$0.00
15	\$373.19	25	\$398.19	\$11.95 Q5
16	\$410.13	25	\$435.13	\$0.00
17	\$435.13	25	\$460.13	\$0.00
18	\$460.13	25	\$485.13	\$14.55 Q6
19	\$499.68	25	\$524.68	\$0.00
20	\$524.68	25	\$549.68	\$0.00
21	\$549.68	25	\$574.68	\$17.24 Q7
22	\$591.93	25	\$616.93	\$0.00
23	\$616.93	25	\$641.93	\$0.00
24	\$641.93	25	\$666.93	\$20.01 Q8
25	\$686.93	25	\$711.93	\$0.00
26	\$711.93	25	\$736.93	\$0.00
27	\$736.93	25	\$761.93	\$22.86 Q9
28	\$784.79	25	\$809.79	\$0.00
29	\$809.79	25	\$834.79	\$0.00
30	\$834.79	25	\$859.79	\$25.79 Q10

0 \$6,500.00  
1 \$6,662.50  
2 \$6,829.06  
6 \$7,538.01

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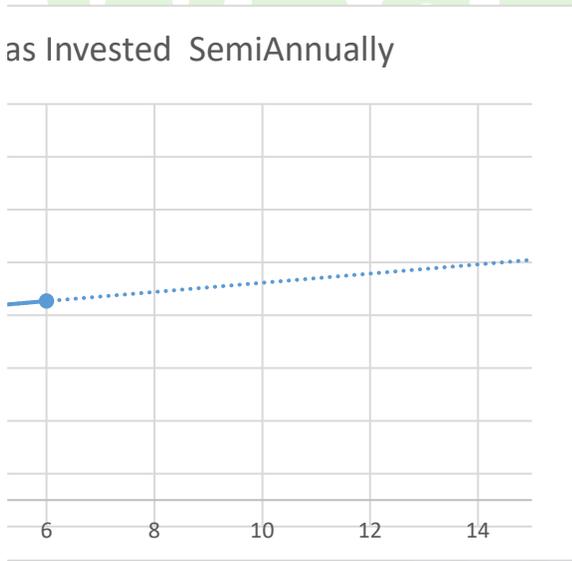
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When rate and compounding periods do NOT coincide (two alternatives) (From [www.frickcpa.com](http://www.frickcpa.com))

### a. Rate Equivalence Approach

Under this approach we convert the rate used for compounding into an equivalent rate based on the payment frequency.

For example, consider our original example where we are making payments annually but compounding monthly.

What annual rate is equivalent to 9% compounded monthly?

The following equation takes the 9% annual rate and converts it to an annual effective rate under monthly compounding...

$$i_{\text{equivalent}} = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.0938$$

In other words, 9% compounded monthly is equivalent to 9.38% compounded annually.

Now we can perform our **FV** of an annuity calculation using the equivalent annual rate...

$$FV = 25 \left( \frac{(1 + 0.0938)^3 - 1}{0.0938} \right) = 82.2616$$

### b. Deconstruction Approach

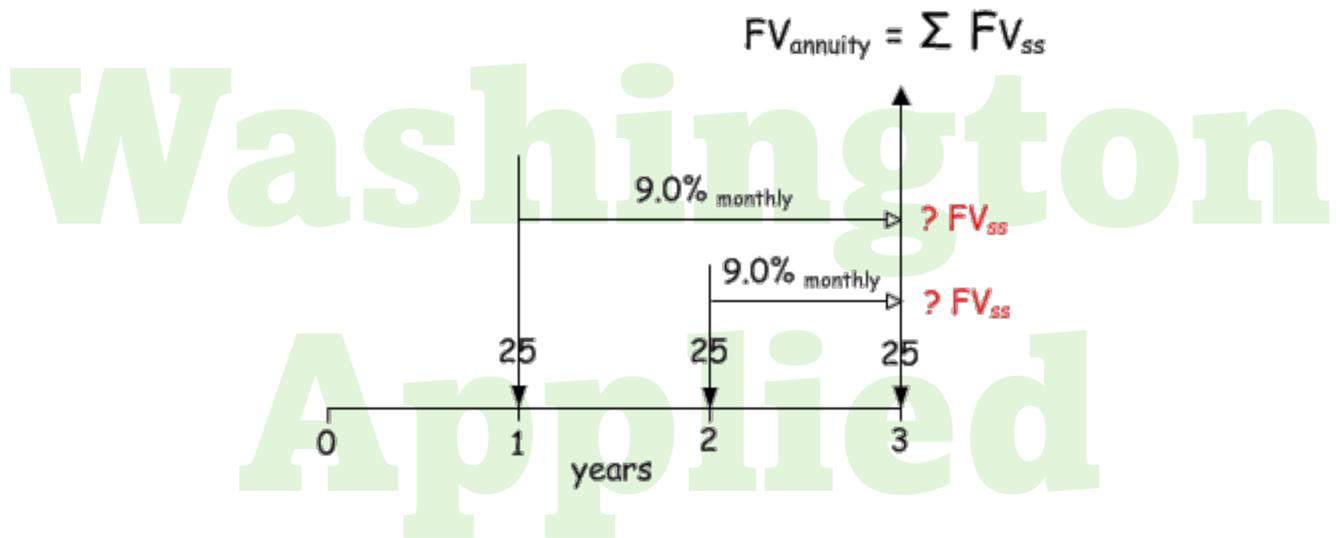
As an alternative to the rate equivalence approach, we can compute the **FV** for each payment and the summation of all of these individual values will be the **FV** of the annuity.

Typically this approach is used when the payment amounts are not equal or the interval between payment dates varies. However, it can also be applied to standard annuities.

Using our original example, the **FV** of a series of three *annual* payments of \$25 at 9% *monthly* compounding is computed as the sum of the **FVs** of three single sum payments of \$25 each with terms of 2, 1 and 0 years.

Graphically this approach looks like this...

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...and crunching the numbers...

$$FV_2 = PV(1+i)^n = 25 \left( 1 + \frac{0.09}{12} \right)^{12(2)} = 29.91$$

$$FV_1 = PV(1+i)^n = 25 \left( 1 + \frac{0.09}{12} \right)^{12(1)} = 27.35$$

$$FV_0 = PV(1+i)^n = 25 \left( 1 + \frac{0.09}{12} \right)^0 = 25.00$$

$$FV_{\text{annuity}} = \sum_{j=0}^2 FV_j = 82.26$$

..we find that our calculated FV of \$82.26 is the same as it was under the rate equivalence approach.

OR (Option 3)

Go to the online calculator to check your work.

<http://draketechnologies.com/LoadQAce.php3?r=qAceSpectrum&f=Calculators/Compounding.htm&l=>

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# WAMC Lesson Plan

Name(s): Kari L. Toms, ktoms@eagles.edu

Lesson Title: 3-7 Future Value of Investments

Date:

Text: Financial Algebra, 2<sup>nd</sup> Edition, Gerver/Srgoi

Lesson Length: 1 periods (55 min)

Domain: Interpreting Functions

Big Idea (Cluster): Interpret functions that arise in applications in terms of context

Common Core State Standards: F-IF8b

Mathematical Practice(s): Functions, graphing

Content Objectives: Calculate FV

Language Objectives:

Vocabulary: Periodic Investment, Rates

Connections Prior to Learning

Questions to Develop Mathematical Thinking:

Common Misconceptions:

- Does this function increase or decrease the value of investment

- Savings the best place to grow money
- You don't need to start saving for retirement until you're in your 40s (bring back compound concepts)

Assessment (Formative and Summative):

- Formative: vocabulary worksheet comprehension and equation graphing; daily check-out
- Summative: Quiz after both single deposit and periodic deposit investment, showing both formulas, substitution, and graphed solutions.

Materials:

- 3-7 Worksheet 1 (Single Deposit Investments)
- 3-7 Worksheet 2 (Periodic Deposit Investments)
- MS Excel
- Scientific Calculator

Instruction Plan: (Day 1)

Launch: 7minute video from St. Louis Federal Reserve about Compound Interest (opt)

Explore: *Warm-up questions on board – then Worksheet 3-7 on own.*

*ASK: By a show of hands, how many of you are saving money towards a goal? For what? Car? Trip? Allow students to respond. Okay, hands down. How many of you know exactly when you're going to reach your savings goal? We are going to calculate the future values of money invested, so we can see how much it will be worth and whether or not we're going to meet those goals.*

*WRITE ON BOARD: Learning Target for the Day: I can calculate future values of single deposit investments and predict values for future dates.*

WARM UP (~10 minutes)

In order to define these terms, we are going to calculate them in action to explain the meaning of these words, before we use them in future value formulae.

PERIODIC DEPOSITS:

Periodic deposits are the number of times that a deposit will repeated during a year. To demonstrate this idea, answer the following questions:

1. What number of times in a year will you deposit money, if you make a trip to the bank:
  - a. Weekly?
  - b. Bi-Weekly?
  - c. Semi-monthly?
  - d. Monthly?
  - e. Quarterly?
  - f. Annually?

## WAMC Lesson Plan

This will represent  $n$  in our formulae, or the number of times an investment is periodically made.

### RATE AS A DECIMAL

Rates are represented as decimals in this formula. When we convert a percentage to a decimal, be sure to drop the percentage symbol and move the decimal two number places to the left.

2. What decimals would be used to represent the following rates (percentages) offered by an investment:
- 3.25%
  - 1.45%
  - 8.15%
  - 7.5%
  - .5%

### FV OF A SINGLE DEPOSIT

The single deposit formula can be used to find the future value of an investment.

$$FV = P(1 + r/n)^{nt}$$

This formula can be simplified, by calculating these items before substituting numbers for the variables.

- the rate per period ( $r/n$ )
- number of compounding periods ( $nt$ )

*A problem may present itself like this:*

Miriam has a savings account with a \$4,800 balance. She has an opportunity to invest it in an account that pays 8.25%, compounded semi-annually. What will that account be worth 3 years in the future?

### STRATEGY

First, underline any words that will help us solve the problem.

Then, determine the rate per period

Next, determine the number of compounding periods.

Last, insert values into the formula (substitute)

One more together, before we break into groups:

Edgar has been awarded a \$10,000 cash scholarship award. He'd like to save it to use for university in two years, which is more expensive than his community college. If he can invest it in an account that will pay him a 6% a year, with interest compounded monthly, what will his account be worth in two years?

Underline.

Rate/Period

Compound Periods?

Values

Allow 25 minutes for students to calculate pairs and enter a scatter plot with trendline into MS Excel for consideration. If incomplete, homework will be necessary.

When I observe students: Verify formatively that they are de-constructing problems, underlining essential items and restructuring items into formula properly. For the last 5-7 minutes of class, check off on your roster of students who has been able to create the coordinate pairs, (80% done), scatter plot & trendline (90% done), and make predictions that can be proved mathematically (100% done).

Questions to Develop Mathematical Thinking as you observe:

A) How many times will they get free money?

B) Is there an exponential advantage to investing early?

Answers:

A) Answers vary, but should indicate the number of periodic compounds.

## WAMC Lesson Plan

B) Advantages include earning interest on interest for additional “free money”

Summarize: Can students see the exponential effect of compounding on single investments? Discuss the growth. For extension, students may flip the equation into a division of principal (as opposed to multiplication) to set targets for savings growth. For new learners, the project may be initially scaffolded with table factors pre-determined to demonstrate the principle with a simpler approach. Administer Quiz at beginning of Day 2, prior to discussing next future value concept.

Career Application(s):

- Investing percentages of income earned into investment accounts
- Saving income for retirement
- Amortization of loans
- Setting savings goals and using sinking funds to find a suitable savings goal using the power of compounding for future values.

21<sup>st</sup> Century Skills and Interdisciplinary Themes:

21<sup>st</sup> Century Interdisciplinary themes (Check those that apply to the above activity.)

- Global Awareness     
  Financial/Economic/Business/Entrepreneurial Literacy     
  Civic Literacy  
 Health/Safety Literacy     
  Environmental Literacy

21<sup>st</sup> Century Skills (Check those that students will demonstrate in the above activity.)

**LEARNING AND INNOVATION**

Creativity and Innovation

- Think Creatively  
 Work Creatively with Others  
 Implement Innovations

Critical Thinking and Problem Solving

- Reason Effectively  
 Use Systems Thinking  
 Make Judgments and Decisions  
 Solve Problems

Communication and Collaboration

- Communicate Clearly  
 Collaborate with Others

**INFORMATION, MEDIA & TECHNOLOGY SKILLS**

Information Literacy

- Access and Evaluate Information  
 Use and manage Information

Media Literacy

- Analyze Media  
 Create Media Products  
Information, Communications and Technology (ICT Literacy)  
 Apply Technology Effectively

**LIFE & CAREER SKILLS**

Flexibility and Adaptability

- Adapt to Change  
 Be Flexible

Initiative and Self-Direction

- Manage Goals and Time  
 Work Independently  
 Be Self-Directed Learners

Social and Cross-Cultural

- Interact Effectively with Others  
 Work Effectively in Diverse Teams

**Productivity and Accountability**

- Manage Projects  
 Produce Results

Leadership and Responsibility

- Guide and Lead Others  
 Be Responsible to Others

**Scaffolding for newer learners: Use table factors for periods.**

**Extension for advanced learners: Utilize amortization and sinking fund problems for present and future values over time. Graph account values in sinking funds and amortization.**

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Chapter 3-7 Problems

1. In reference to Ron Lewis' quote, the never too early reference applies to the snowballing effect that is derived from compound interest. The power in that compounding is best illustrated when shown at an early age, over time.

2.  $1,000(1+(.05/1))^{(4*1)}=FV$   $1,000(1.05)^4=1,215.51$

3.  $500(1+(.03/2))^{(2*2)}=FV$   $500(1.015)^4=530.68$

4.  $10,000(1+(.04/4))^{(3*4)}=FV$   $10,000(1.01)^{12}=1,268.25$

5.  $\frac{200((1+(.045/12))^{(12*4)}-1)}{.045/12}$   $\frac{200((1+(.045/12))^{(12*4)}-1)}{.045/12}$  =10,496.75

\*\*\* (book is one cent higher, but perhaps incorrect) Even when stored to hundred-thousandths, accuracy is compromised.

6.  $\frac{100((1+(.024/52))^{(52*3)}-1)}{.024/52}$   $\frac{100((1+(.024/52))^{(52*3)}-1)}{.024/52}$  =16,171.49

\*\*\*Book is listing three cents less. Problems should indicate to which number place to use, in order to allow accurate answers while calculating.

7.  $\frac{600((1+(.05/4))^{(5*4)}-1)}{.05/4}$   $\frac{600((1+(.05/4))^{(5*4)}-1)}{.05/4}$  =13,537.79

8.  $2000((1+(.035/2))^{(16*2)})$   $2000((1+(.035/2))^{(16*2)})$  =3484.43

Sydney 9. $\frac{100((1+(.05/12))^{(12*10)}-1)}{.05/12}$ <b>15,528.23</b>	Benny $\frac{80((1+(.08/12))^{(12*10)}-1)}{.08/12}$ = <b>14,635.68</b>
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**If 20 years are considered:**

$\frac{100((1+(.05/12))^{(12*20)}-1)}{.05/12}$ <b>41, 103.04</b>	$\frac{80((1+(.08/12))^{(12*20)}-1)}{.08/12}$ = <b>47,122.10</b>
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**\*\*Text answers are slightly different, as expected with multiple compounds without the number places indicated.**

10.  $=A3*((1+a4/a6)^{(a6*a5)}-1)/(a4/a6)$

11. RULE OF 72

- a.  $72/1=72$  years (book states 70 years)
- b.  $72/2=36$  years (book states 35)
- c.  $72/6=12$  years
- d. Larger numbers

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e.  $72/1.75=41.14\text{years} +10\text{year old} =51\text{years}$

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NAME: \_\_\_\_\_

PERIOD: \_\_\_\_\_

### Chapter 3-7: Calculating Future Value of an Investment

In order to define these terms, we are going to calculate them in action to explain the meaning of these words, before we use them in future value formulae.

#### PERIODIC DEPOSITS:

Periodic deposits are the number of times that a deposit will be repeated during a year. To demonstrate this idea, answer the following questions:

1. What number of times in a year will you deposit money, if you make a trip to the bank:
  - a. Weekly?
  - b. Bi-Weekly?
  - c. Semi-monthly?
  - d. Monthly?
  - e. Quarterly?
  - f. Annually?

This will represent  $n$  in our formulae, or the number of times an investment is periodically made.

#### RATE AS A DECIMAL

Rates are represented as decimals in this formula. When we convert a percentage to a decimal, be sure to drop the percentage symbol and move the decimal two number places to the left.

2. What decimals would be used to represent the following rates (percentages) offered by an investment:
  - a. 3.25%
  - b. 1.45%
  - c. 8.15%
  - d. 7.5%
  - e. .5%

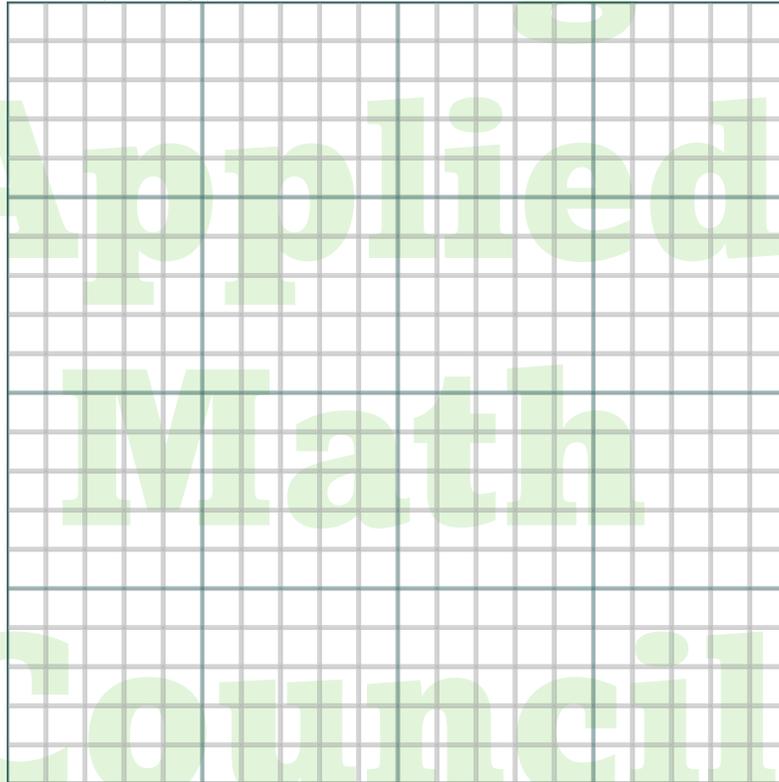
#### FV OF A SINGLE DEPOSIT

The single deposit formula can be used to find the future value of an investment.

$$FV = P(1 + r/n)^{nt}$$

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3. Underline the elements needed in this problem to fulfill the formula listed above:
- Colton earned \$6,500 this summer working. If he invested his income in a Certificate of Deposit account that pays 5% Semi-Annually, how much will Colton have in his account after 6 months? 1 year? 3 years?
  - Graph the growth in his account by re-calculating the future values of his investment with the corresponding periods.



- Using a trend line, can you predict what his investment might be worth in 5 years?
- Using the formula, verify your prediction by calculating a five-year time period for his single deposit.
- Is this a linear function?

## Chapter 3-7: Calculating Future Value of an Investment

In order to define these terms, we are going to calculate them in action to explain the meaning of these words, before we use them in future value formulae.

### PERIODIC DEPOSITS:

Periodic deposits are the number of times that a deposit will be repeated during a year. To demonstrate this idea, answer the following questions:

1. What number of times in a year will you deposit money, if you make a trip to the bank:
  - a. Weekly? **52**
  - b. Bi-Weekly? **26**
  - c. Semi-monthly? **24**
  - d. Monthly? **12**
  - e. Quarterly? **4**
  - f. Semi-Annually? **2**
  - g. Annually? **1**

These periods will represent  $n$  in our formulae, or the number of times an investment is periodically made.

### RATE AS A DECIMAL:

Rates are represented as decimals in this formula. When we convert a percentage to a decimal, be sure to drop the percentage symbol and move the decimal two number places to the left.

2. What decimals would be used to represent the following rates (percentages) offered by an investment:
  - a. 3.25% **.0325**
  - b. 1.45% **.0145**
  - c. 8.15% **.0815**
  - d. 7.5% **.075**
  - e. .5% **.005**

FV OF A SINGLE DEPOSIT

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The single deposit formula can be used to find the future value of an investment.

$$FV = P(1 + r/n)^{nt}$$

*A problem may present itself like this:*

Miriam has a savings account with a \$4,800 balance. She has an opportunity to invest it in an account that pays 8.25%, compounded semi-annually. What will that account be worth 3 years in the future?

STRATEGY

First, underline any words that will help us solve the problem.

Then, determine the rate per period **.0825/2=.04125**

Next, determine the number of compounding periods. **3years\*2semi-annual periods = 6 per**

Last, insert values into the formula (substitute)

$$=4800(1.04125)^6$$

$$=6117.46$$

One more together, before we break into groups:

Edgar has been awarded a \$10,000 cash scholarship award. He'd like to save it to use for university in two years, which is more expensive than his community college. If he can invest it in an account that will pay him a 6% a year, with interest compounded monthly, what will his account be worth in two years?

Underline.

Rate/Period **.06/12=.005**

Compound Periods? **2years\*12 months = 24 periods**

Values

$$=10,000(1.005)^{24}$$

$$=11,271.60$$

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NAME: \_\_\_\_\_

PERIOD: \_\_\_\_\_

### Chapter 3-7: Calculating Future Value of an Investment

RATE PER PERIOD:

1. What will be the rate per period, if the nominal rate is 12%
  - a. Monthly?
  - b. Quarterly?
  - c. Semi Annually?
2. What will be the rate per period, if the nominal rate is 7%
  - a. Monthly?
  - b. Annually?
3. What will be the rate per period, if the nominal rate is 3.45%
  - a. Monthly?
  - b. Semi-Annually?

COMPOUNDING PERIODS:

4. How many periods will an investment compound in three years, if interest is added:
  - a. Quarterly?
  - b. Monthly?
5. How many periods will an investment compound in five years, if interest is added:
  - a. Quarterly?
  - b. Semi-Annually?
  - c. Annually?
6. Underline the elements needed in this problem to fulfill the formula listed above:
  - a. Colton earned \$6,500 this summer working. If he invested his income in a Certificate of Deposit account that pays 5% Semi-Annually, how much will Colton have in his account after 6 months? 1 year? 3 years?
  - b. In MS Excel, graph the growth, using a scatter plot style, in his account by re-calculating the future values of his investment with the corresponding periods.
  - c. Add a trend line on your scatter plot. Using that trend line, can you predict what his investment might be worth in 5 years?
  - d. Using the formula, verify your prediction by calculating a five-year time period for his single deposit.
  - e. Is this a linear function?

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## Chapter 3-7: Calculating Future Value of an Investment

### RATE PER PERIOD:

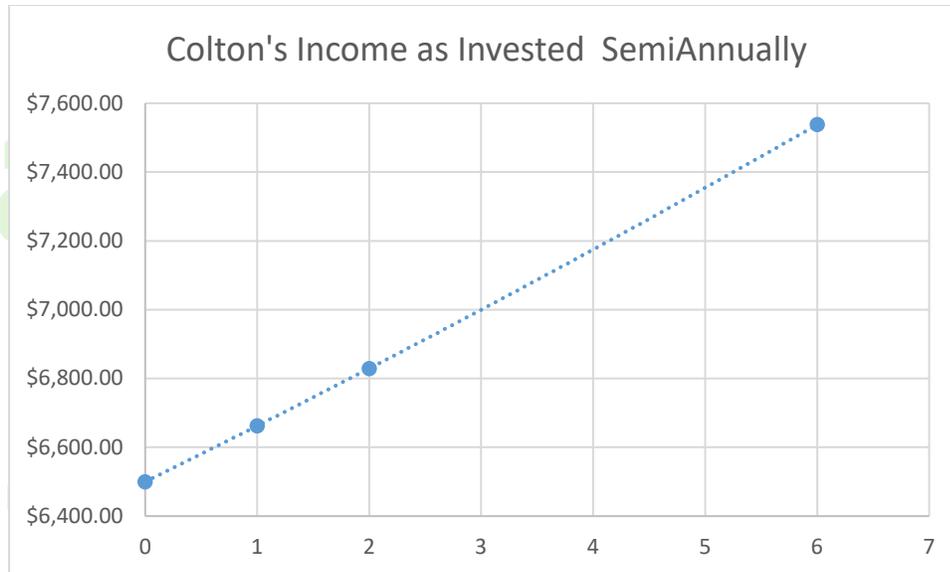
1. What will be the rate per period, if the nominal rate is 12%
  - a. Monthly?  $.12/12=.01$
  - b. Quarterly?  $.12/4=.03$
  - c. Semi Annually?  $.12/2=.06$
2. What will be the rate per period, if the nominal rate is 7%
  - a. Monthly?  $.07/12=.005833$
  - b. Annually?  $.07/1=.07$
3. What will be the rate per period, if the nominal rate is 3.45%
  - a. Monthly?  $.0345/12=.002875$
  - b. Semi-Annually?  $.0345/2=.01725$

### COMPOUNDING PERIODS:

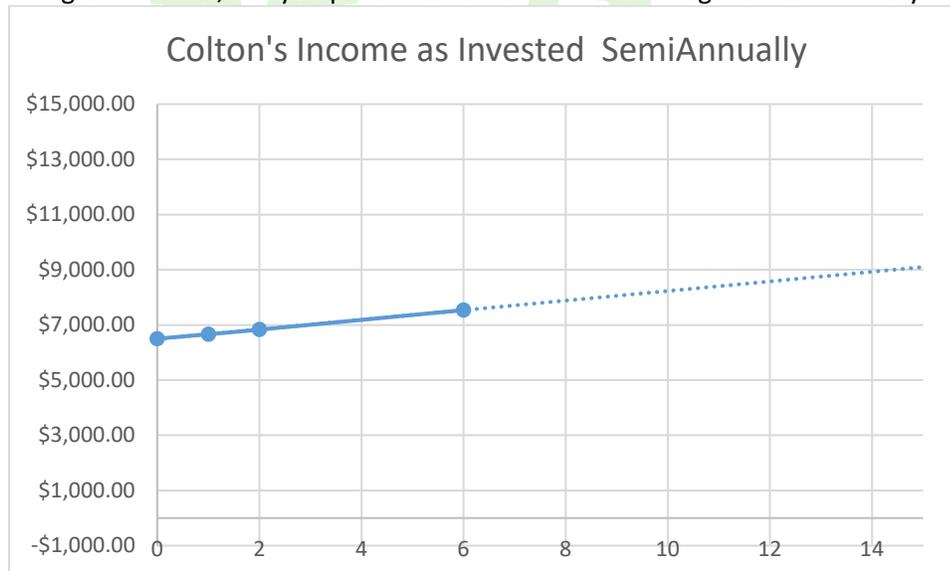
4. How many periods will an investment compound in three years, if interest is added:
  - a. Quarterly? **3 years \* 4 quarters = 12 periods**
  - b. Monthly? **3 years \* 12 months = 36 periods**
5. How many periods will an investment compound in five years, if interest is added:
  - a. Quarterly? **5 years \* 4 quarters = 20 periods**
  - b. Semi-Annually? **5 years \* 2 semi-annual periods = 10 periods**
  - c. Annually? **5 years \* 1 annual period = 5 periods**

1. Underline the elements needed in this problem to fulfill the formula listed above:
  - a. Colton earned \$6,500 this summer working. If he invested his income in a Certificate of Deposit account that pays 5% Semi-Annually, how much will Colton have in his account after 6 months? 1 year? 3 years?  
\$6,500 will be "P" or principal in the formula  
"R" will substituted with the 5% (converted to a decimal for rate of .05)  
"N" in the formula will be substituted with 2 for Semi-Annual periods  
"T" will be the number of years. Initially, the first calculation has .5 (half) a year for the semi-annual period. Subsequent calculations will use 1 year, 3, years, etc.
  - b. Create a list of paired data showing the number of compounding semi-annually periods with the future value.
    - i. 1 period (six months time) 1, 6662.50
    - ii. 2 periods (1 year) 2, 6829.06
    - iii. 6 periods (3 years) 6, 7538.01
  - c. Graph the growth in his account by re-calculating the future values of his investment with the corresponding periods.

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- d. Using a trend line, can you predict what his investment might be worth in 5 years?



Approximately \$8,600? A reasonable prediction arrived by adding trend line in MS Excel.

- e. Using the formula, verify your prediction by calculating a five-year time period for his single deposit.

10 periods: (5 years) \$8,320.55

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**RESOURCES:**

[http://www.stlouisfed.org/education\\_resources/no-frills-money-skills/episode-1-growing-money/](http://www.stlouisfed.org/education_resources/no-frills-money-skills/episode-1-growing-money/)

# Washington Applied Math Council

<https://wa-appliedmath.org/>

NAME: \_\_\_\_\_

PERIOD: \_\_\_\_\_

### Chapter 3-7: SECTION QUIZ, Future Values

*Directions: Based on yesterday's exploration of future values, answer the following questions.*

1. What number of times in a year will you earn interest:
  - a. Semi-monthly?
  - b. Quarterly?
2. What decimals would be used to represent the following rates (percentages) offered by an investment:
  - a. 4.75%
  - b. 1.15%
3. Underline the elements needed in this problem and solve for future value:
  - a. Joseph earned \$4,300 this summer working. If he invested his income in a money market account that pays 6% quarterly, how much will Joe have in his account after 1 year?
4. At age 11, you have inherited an insurance policy worth \$200,000. At this age, you don't need the money, but you have been advised to invest the money to buy a house in the future. You can invest in an account that yields 6% nominally, which is compounded monthly. If you leave it until the age of 25, how much money will that account hold?

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NAME: \_\_\_\_\_

PERIOD: \_\_\_\_\_

### Chapter 3-7: SECTION QUIZ, Future Values

Directions: Based on yesterday's exploration of future values, answer the following questions.

1. What number of times in a year will you earn interest:
  - a. Semi-monthly? **24**
  - b. Quarterly? **4**
2. What decimals would be used to represent the following rates (percentages) offered by an investment:
  - a. 4.75% **.0475**
  - b. 1.15% **.0115**
3. Underline the elements needed in this problem and solve for future value:
  - a. Joseph earned \$4,300 this summer working. If he invested his income in a money market account that pays 6% quarterly, how much will Joe have in his account after 1 year?

$$\begin{aligned} & 4,300(1+r/n)rt \\ & 4,300(1+(.06/4))^{(4*1)} \\ & 4,300(1+.015)^{(4)} \\ & 4,300(1.015)^{(4)}=\$4563.86 \end{aligned}$$

4. At age 11, you have inherited an insurance policy worth \$200,000. At this age, you don't need the money, but you have been advised to invest the money to buy a house in the future. With your present value of \$200,000, you can invest in an account that yields 6% nominally, which is compounded monthly. If you leave it until the age of 25, how much money will that account hold?

$$\begin{aligned} & 200,000(1+r/n)rt \\ & 200,000(1+(.06/12))^{(12*14)} \\ & 200,000(1+.005)^{(168)} \\ & 200,000(1.005)^{(168)}=\$462,304.77 \end{aligned}$$

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