

WAMC Lab Template

Math Concept(s):

Source / Text: Financial Algebra (Cengage, 2nd edition)

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Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

This lab will be in the beginning of the lesson to show the value of early investing. Students will present a visual display of the difference between 2 individuals who start a job at the same age and choose to start investing at different ages. They will use a desmos calculator, creating tables, and using excel to graph the return.

Lab Plan

Lab Title: The Retirement Pool

Prerequisite skills: knowledge of future value formula, knowledge of percentage and conversion to decimal form and ability to use table and create a graph in excel.

Lab objective: The objective of this lab is for students to realize the benefits of starting a retirement investment account early in career. They will graph the results of various scenarios and recognize exponential growth.

Standards: (Note SPECIFIC relationship to Science, Technology, and/or Engineering)

Mathematics K–12 Learning Standards:

- A-SSE
- A-CED
- F-IF
- F-BF
- S-MD

Standards for Mathematical Practice:

- 1, 3, 4, 5, 6, 7

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- RST.11-12.7

K-12 Science Standards

-

Technology

- 1d, 4a, 6c, 7c

Engineering

Leadership/21st Century Skills:

| | | | |
|--|--|--|---|
| <p><u>21st Century Interdisciplinary themes</u> (Check those that apply to the above activity.)</p> <input type="checkbox"/> Global Awareness <input checked="" type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy <input checked="" type="checkbox"/> Civic Literacy <input type="checkbox"/> Health/Safety Literacy <input type="checkbox"/> Environmental Literacy | | | |
| <p><u>21st Century Skills</u> (Check those that students will demonstrate in the above activity.)</p> | | | |
| <p>LEARNING AND INNOVATION</p> <p><u>Creativity and Innovation</u></p> <input type="checkbox"/> Think Creatively <input type="checkbox"/> Work Creatively with Others <input type="checkbox"/> Implement Innovations <p><u>Critical Thinking and Problem Solving</u></p> <input checked="" type="checkbox"/> Reason Effectively <input checked="" type="checkbox"/> Use Systems Thinking <input type="checkbox"/> Make Judgments and Decisions <input checked="" type="checkbox"/> Solve Problems <p><u>Communication and Collaboration</u></p> <input checked="" type="checkbox"/> Communicate Clearly <input checked="" type="checkbox"/> Collaborate with Others | <p>INFORMATION, MEDIA & TECHNOLOGY SKILLS</p> <p><u>Information Literacy</u></p> <input checked="" type="checkbox"/> Access and Evaluate Information <input checked="" type="checkbox"/> Use and manage Information <p><u>Media Literacy</u></p> <input type="checkbox"/> Analyze Media <input type="checkbox"/> Create Media Products <p><u>Information, Communications and Technology (ICT Literacy)</u></p> <input checked="" type="checkbox"/> Apply Technology Effectively | <p>LIFE & CAREER SKILLS</p> <p><u>Flexibility and Adaptability</u></p> <input type="checkbox"/> Adapt to Change <input type="checkbox"/> Be Flexible <p><u>Initiative and Self-Direction</u></p> <input checked="" type="checkbox"/> Manage Goals and Time <input checked="" type="checkbox"/> Work Independently <input checked="" type="checkbox"/> Be Self-Directed Learners <p><u>Social and Cross-Cultural</u></p> <input checked="" type="checkbox"/> Interact Effectively with Others <input type="checkbox"/> Work Effectively in Diverse Teams | <p>Productivity and Accountability</p> <input type="checkbox"/> Manage Projects <input type="checkbox"/> Produce Results <p>Leadership and Responsibility</p> <input type="checkbox"/> Guide and Lead Others <input checked="" type="checkbox"/> Be Responsible to Others |

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Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- 2 glass jars of the same size per group
- Gallon pitchers of water per group
- Quarter cup measuring cup per group
- Desmos calculator per student

Set-Up Required:

- Have students divide into groups of 2-3
- Review the formula for finding future growth
-

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

-

Cooperative Learning:

- Students will work in groups complete a set of scenarios that model saving for retirement by periodic contributions to an investment account.
-

Expectations:

Students will make estimates of the amount of time it will take to save \$1,000,000 for retirement. They will be given the opportunity to try several scenarios to prove or disprove their estimate.

Timeline:

- 10 min – Review future value formula and using Desmos to enter the formula.
- 15 min – Complete the first section of worksheet using the glass jars and water.
- 15 min – Use Desmos graph to identify exponential growth.
- 10 min – Use sliders in Desmos to answer the section questions in the final section of worksheet.
- 10 min – Work together to present best time to start investing to have \$1,000,000 by the age of 65.

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- To have a visual reference for what periodic investments will make in the long run using compounded interest and future value.

Career Applications

- All citizens, Bankers, Stock Brokers, Investment Analyst

Optional or Extension Activities

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The Retirement Pool

Picture your retirement years. Would you like to be young enough to travel and enjoy your hobbies or work until you are in your 80's? How much do you think you will need to save to have the ability to retire by age 60? How much will you need to save each pay period to achieve this goal? This lab will guide you through finding the answers to these questions.

Section 1:

With your group, discuss the questions and agree on one answer per group.

Robin and Taylor are the same age and starting a new job together. They are discussing the best investment strategy. They want to be able to retire with \$1,000,000 in the account. Help them figure out which is the better plan.

Each group will have 2 glass jars with a tape measure attached to the back side with 0 end at the bottom of the jar, and a pitcher of water.

Calculate the following scenarios:

Taylor is planning to start contributing \$250 to a retirement investment account at the age of 23. How much will be in the account that pays 11% compounded monthly at the age of 30?

Robin is planning to wait to start contributing \$300 to a retirement account until the age of 30. The account pays 11% compounded monthly. What will be the balance of the account at the age of 30?

Use the Future Value formula in Desmos to find the balance for each person at 30 years old.

For every \$20000 in each account, make the level in the jar should be .25 inch.

Calculate the balance of the accounts at age 35. And adjust the level of water in the jars.

Take a minute to discuss in your group whether you think it is realistic for them to want \$1,000,000 by age 65? Estimate the age at which each person will reach \$1,000,000.

Create a table of the data for ages 35-65 and transfer the information to an Excel spreadsheet to graph the growth of each account.

Add water to the jars for each level to and compare the results. Which pool would you rather dive into?

Who has the better plan? Will either of them reach \$1,000,000 by the age of 65?

Are there other ways for them to get there? Or to get there before age 65?

If Taylor quit contributing to the account, would he still have \$1,000,000? How could you determine that?

TEACHERS – the answers for the table and graph are in attached excel file of same name. There is also a Future value of periodic investment excel file that they can put information into.

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Use the financial formula to complete the following calculations

What will be the future value of contributing \$500 every month for 30 years at 6%.

How much will have to be contributed monthly to have \$1,200,000 in the retirement account in 30 years?

Ron has a 401(k) plan provided by his employer who will match 50% up to 7% of his salary. The company allow him to make contributions up to 12% of his salary. He plans on contributing 10% of his salary. Calculate how much his employer will contribute and if he could do anything to increase that amount. Show all you work

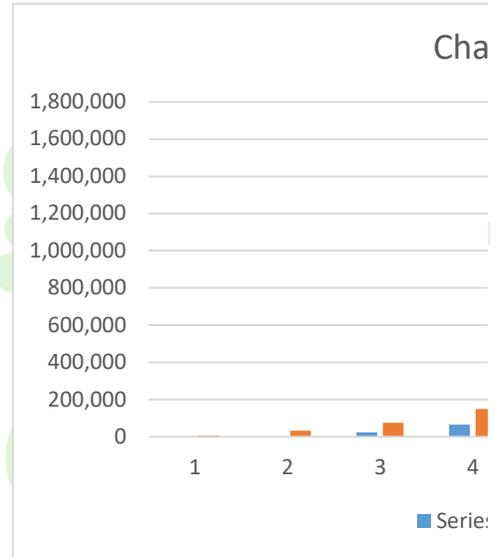
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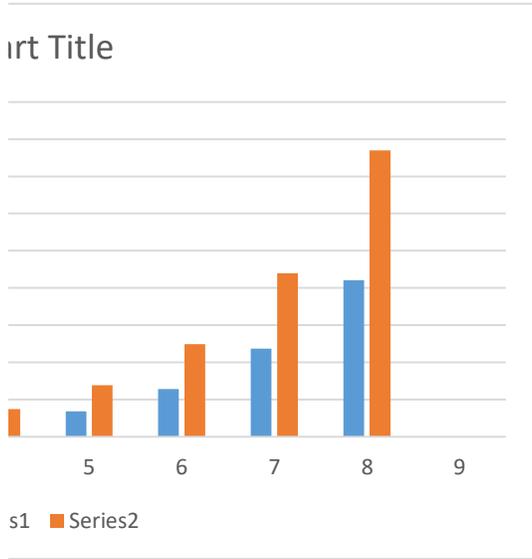
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| Age | Robin | Taylor | |
|-----|--------|---------|----|
| 25 | 0 | 6677 | 2 |
| 30 | 0 | 31423 | 7 |
| 35 | 23855 | 74208 | 12 |
| 40 | 65099 | 148179 | 17 |
| 45 | 136407 | 276069 | 22 |
| 50 | 256691 | 497181 | 27 |
| 55 | 472839 | 879464 | 32 |
| 60 | 841355 | 1540398 | 37 |
| 65 | | | |



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WAMC Lesson Plan

Name(s): Maureen Moore

Email Address: maureen.moore@wvwd.org

Lesson Title: FA10.1 How Deep is your Retirement Pool?

Date: June 26, 2019

Text: Cengage Financial Algebra STEM Correlation: T, M Lesson Length: 2 days

| | |
|--|---|
| Big Idea (Cluster): Understand solving equations as a process of reasoning and explain the | |
| Mathematics K–12 Learning Standards A-SSE 1a and 1b F-IF: 1, 2, 7 F-LE: 5 A-REI: 1 A-CED: 4 | |
| Mathematical Practice(s) 1, 2, 4, 5, 6: | |
| <p>Content Objectives:</p> <ul style="list-style-type: none"> • Calculate future values of retirement investments of both single deposit and periodic. • Compare tax savings by making contributions to pre-tax retirement savings accounts. • Calculate an employer's matching contribution to an investment account. | <p>Language Objectives (ELL):</p> <p>RST.11-12.7</p> |
| <p>Vocabulary:</p> <ul style="list-style-type: none"> • Retirement • Semi-retired • Pre-tax dollars • After-tax investments • Individual Retirement Account (IRA) • Traditional IRA • Tax-deferred • Roth IRA • Tax-exempt • 401(k) • SEP plan • SIMPLE plan • 403(b) | <p>Connections to Prior Learning</p> <ul style="list-style-type: none"> • Future value of periodic investment • Present value of periodic investment • Using exponential functions • Compare data from a table • Calculating percent of a number |
| <p>Questions to Develop Mathematical Thinking:</p> <ul style="list-style-type: none"> • If you want to retire by age 65, how much money do you need to have saved? • How much do you need to save monthly to meet that goal? • Would you be willing to make higher contributions to a retirement account | <p>Common Misconceptions:</p> <ul style="list-style-type: none"> • You should wait until you are financially stable to save for retirement. • You can't afford to save for retirement because I have to pay for kid's college expenses. • My employer will provide a retirement savings plan that will be enough for me to retire comfortably. |

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to retire before 65? If so, what kinds of options do you have?

- Is a Roth IRA better than a traditional IRA?

- Social Security will take care of your retirement.

Assessment (Formative and Summative):

- Formative – Teacher will observe and evaluate knowledge and participation while students participate in a group discussion about best plan
- Assignment of text questions.
- Summative: Short quiz over concepts

Materials:

- Financial Algebra (Cengage) textbook 2nd edition
- Calculator (financial)
- Worksheet with scenarios

Instruction Plan:

Introduction: Present retirement savings options using pre-tax dollars, after-tax dollars and investment options

Explore:

- How do you calculate how much you will have saved for retirement by age 65?
- How can I maximize the value of my retirement account?
- When should you open a retirement savings account?
- How do you choose the best type of investment?
- Teacher will go through vocabulary list and how the terms retirement savings calculations.
- Teacher will guide students through examples in the book.

When I observe students:

- Encourage correct use of vocabulary.
- Watch for correct use of parentheses when entering expressions into the calculator.

Questions to Develop Mathematical Thinking as you observe:

- What information do you need to solve the problem?
- What formulas did you use?
- Does your answer seem reasonable?

Answers:

- You will need all data for Future Value and Present Value formulas.

Summarize:

Day 1

After opening questions, student will discuss the importance of having a retirement savings account.

Review vocabulary terms with students

Review the formulas for Future Value of periodic investments and review the variables.

Work through the example problem with the students with emphasis on how to enter the data into the calculator.

Review the formula for Present Value of periodic investments and have students work

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through example problem in book.

Assign problems 1 and 2 from the applications on page 614 to do on their own.

Take Quiz 1 in class.

Day 2 Discuss options for pre-tax dollars and after-tax dollars for retirement savings account.

Discuss difference between after-tax and pre-tax dollars. How do they affect the amount of the ending balance? How do they affect the income tax on withdrawal?

Review using tax tables with students and have them find tax savings for example on worksheet.

Discuss the penalties for making an early withdrawal from an IRA and assign worksheet problems.

Discuss the advantages of maxing out your employers matching 401(k) funds. Assign problems 5 and 6 from applications in book.

Take Quiz 2 in class

Homework: Problems 8-14

Career Application(s):

- Investment broker, Financial Planner,

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

- | | | |
|---|--|---|
| <input type="checkbox"/> Global Awareness | <input checked="" type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy | <input type="checkbox"/> Civic Literacy |
| <input type="checkbox"/> Health/Safety Literacy | <input type="checkbox"/> Environmental Literacy | |

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

Creativity and Innovation

- Think Creatively
- Work Creatively with Others
- Implement Innovations

Critical Thinking and Problem Solving

- Reason Effectively
- Use Systems Thinking
- Make Judgments and Decisions
- Solve Problems

Communication and Collaboration

- Communicate Clearly
- Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

Information Literacy

- Access and Evaluate Information
- Use and manage Information

Media Literacy

- Analyze Media
 - Create Media Products
- Information, Communications and Technology (ICT Literacy)
- Apply Technology Effectively

LIFE & CAREER SKILLS

Flexibility and Adaptability

- Adapt to Change

- Be Flexible

Initiative and Self-Direction

- Manage Goals and Time

- Work Independently

Social and Cross-Cultural

- Interact Effectively with Others

- Work Effectively in Diverse Teams

Productivity and Accountability

- Manage Projects

- Produce Results

Leadership and Responsibility

- Guide and Lead Others

- Be Responsible to Others

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Retirement Savings Account Balance Quiz 1

1. Identify the variables in this formula.

$$B = \frac{P \left(\left(1 + \frac{r}{n} \right)^{nt} \right)}{\frac{r}{n}}$$

- a. $B =$
- b. $P =$
- c. $r =$
- d. $n =$
- e. $t =$
2. Adam is 35 years old working as a retail manager and would like to retire in 20 years. He has been making monthly deposits of \$800 in an IRA with an APR of 4.0% compounded monthly. What will the for balance be in 20 years. Show your work.
3. Adam wants to have \$50,000 in his retirement account by age 55. Will his plan get him to that balance? If not, how long will he have to work to get to \$750,000? Show your work. (Hint: think about your formula)

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Retirement Savings Account Balance Quiz 2

Vocabulary

- Pre-tax dollars
- After-tax investments
- Traditional IRA
- Tax-deferred
- Roth IRA
- 401(k)
- 403(b)

Nelson makes \$120,000 per year. His employer offers a 401(k) plan in which they will match 40% of his contributions up to a maximum of 7% of his annual salary. His employer allows contributions up to a maximum of 15% Nelson's salary per year. If Nelson contributes \$200 of each biweekly paycheck, how much will his employer contribute to his 401(k)? Show your work on separate page.

John is 60 years old. He plans to retire in 2 years. He now has \$400,000 in a savings account that yields 2.1% interest compounded monthly. He has calculated that his final working year's salary will be \$88,000. He has been told by his financial advisor that he should have 60-70% of his final year's annual income available for use in each year when he retires.

- What is the range of income that his financial advisor thinks he must have per year once he retires?
- Determine how much he will have in his account at the ages of 61 and 62.
- Assume that John is planning to use 65% of his current salary in each of his first 5 years of retirement. What should that annual amount be?
- John has decided that he will need \$2000 each month from his savings account to help him reach his desired annual income during retirement. Will John be able to make withdrawals of \$2000 each month from his savings account for 20 years? Explain your answer. Show your work on separate page.

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FUTURE VALUE - PERIODIC INVESTMENT

| | | |
|------|---|---|
| | B | Ending Balance |
| 100 | p | Monthly Retirement Contribution |
| 0.11 | r | Interest rate expressed as a decimal |
| 12 | n | Number of times interest is compounded annually |
| 10 | t | Number of years |

\$21,699.81

Ending Balance

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| | |
|---|----------------|
| P | Parenthesis |
| E | Exponent |
| M | Multiplication |
| D | Division |
| A | Addition |
| S | Subtraction |

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