

WAMC Lab Template

Math Concept(s): Chapter 10, Section 3 - Pensions

Developed by: Claus Joens
E-Mail: cjoens@concrete.k12.wa.us
Date: Summer Conference 2019

Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

Lab Plan

Lab Title: BUY YOUR LIFE BACK

Prerequisite skills: Interest Rates, Mortgage Calculator, Amortization, Exponential Functions, Compound Interest

Lab objective: Students Identify a Pathway to Retirement through how they purchase a house

Standards: (Note SPECIFIC relationship to Science, Technology, and/or Engineering)

Mathematics K–12 Learning Standards:

- A-APR6, A-CED2, A-CED3, A-REI6, A-SSE1, F-BF1, G-C5, G-MG3, S-ID6a, S-ID6c, S-ID8

Standards for Mathematical Practice:

- Math Practice 3—Construct viable argument and critique reasoning of others.
- Math Practice 4—Model with mathematics (apply mathematics to solve problems arising in everyday life, society and the workplace)
- Math Practice 5—Use appropriate tools and estimate strategically.
- Math Practice 6—Attend to Precision
- Math Practice 8--Look for and express regularity in repeated reasoning

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- SL4 Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.
- SL5 Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.

- SL6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
- RST7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
- L4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 9–10 reading and content, choosing flexibly from a range of strategies.

K-12 Science Standards

- N/A

Technology

- 1.3.2 Locate and organize information from a variety of sources and media.
- 1.3.3 Analyze, synthesize and ethically use information to develop a solution, make informed decisions and report results.
- 2.1.1 Practice personal safety.
- 2.1.2 Practice ethical and respectful behavior.
- 2.4.1 Formulate and synthesize new knowledge.

Engineering

- N/A

Leadership/21st Century Skills:

<u>21st Century Interdisciplinary themes</u> (Check those that apply to the above activity.)			
<input checked="" type="checkbox"/> Global Awareness	<input checked="" type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy	<input type="checkbox"/> Civic Literacy	
<input type="checkbox"/> Health/Safety Literacy	<input type="checkbox"/> Environmental Literacy		
<u>21st Century Skills</u> (Check those that students will demonstrate in the above activity.)			
LEARNING AND INNOVATION	INFORMATION, MEDIA & TECHNOLOGY SKILLS	LIFE & CAREER SKILLS	Productivity and Accountability
<u>Creativity and Innovation</u>	<u>Information Literacy</u>	<u>Flexibility and Adaptability</u>	<u>Manage Projects</u>
<input checked="" type="checkbox"/> Think Creatively	<input checked="" type="checkbox"/> Access and Evaluate Information	<input checked="" type="checkbox"/> Adapt to Change	<input checked="" type="checkbox"/> Produce Results
<input type="checkbox"/> Work Creatively with Others	<input checked="" type="checkbox"/> Use and manage Information	<input checked="" type="checkbox"/> Be Flexible	<input type="checkbox"/> Guide and Lead Others
<input type="checkbox"/> Implement Innovations	<u>Media Literacy</u>	<u>Initiative and Self-Direction</u>	<u>Leadership and Responsibility</u>
<u>Critical Thinking and Problem Solving</u>	<input checked="" type="checkbox"/> Analyze Media	<input checked="" type="checkbox"/> Manage Goals and Time	<input type="checkbox"/> Be Responsible to Others
<input checked="" type="checkbox"/> Reason Effectively	<input type="checkbox"/> Create Media Products	<input checked="" type="checkbox"/> Work Independently	<input type="checkbox"/> Be Responsible to Others
<input checked="" type="checkbox"/> Use Systems Thinking	<u>Information, Communications and Technology (ICT Literacy)</u>	<input checked="" type="checkbox"/> Be Self-Directed Learners	
<input checked="" type="checkbox"/> Make Judgments and Decisions	<input checked="" type="checkbox"/> Apply Technology Effectively	<u>Social and Cross-Cultural</u>	
<input checked="" type="checkbox"/> Solve Problems		<input checked="" type="checkbox"/> Interact Effectively with Others	
<u>Communication and Collaboration</u>		<input type="checkbox"/> Work Effectively in Diverse Teams	
<input checked="" type="checkbox"/> Communicate Clearly			
<input type="checkbox"/> Collaborate with Others			

- 2.D.2 Identify and ask significant questions that clarify various points of view and lead to better solutions

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Lab Instructions

Background: Saving for your Retirement

Go to BLS.Gov and find the average income for your career.....\$ _____

Calculate the average income for all workers in the USA.....\$ _____

Calculate the monthly income based on USA average annual income.....\$ _____

Calculate 15% of the average USA monthly income.....\$ _____

Calculate the amount you would have for retirement if you invested the 15% at a rate of return of 11% for 30 years.....\$ _____

Calculate the amount you would have for retirement if you invested the 15% at a rate of return of 5% for 30 years.....\$ _____

Strategic Analysis

How do you how much you will need for retirement? Since this is your first attempt at retirement planning, we will use a fixed amount. When I graduated high school (1981), the expected amount required was \$750,000.

Questions:

Did 15% of the average US income at 11% return produce \$750,000?

Did 15% of the average US income at 5% return produce \$750,000?

Why is one more or less than the other?

Is your chosen career expected to pay more or less per year than the US average?

How will that impact saving for your retirement?

If you have less than \$750,000, how will you make up the difference?

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The Biggest Investment

For most people, their largest investment is their house. Does every country look at housing the same way? Does every country finance housing the same way? What choice do you have?

The average size house in Europe is 1,400 square feet (think double wide mobile home). The average size house in the USA is 2,200 square feet.

Find a house in the county near where you think you will find a job (within 15 miles).

What is the price of a 1,400 square foot house (+/- 100 square feet).....\$ _____
(Save a PDF one page copy of the real estate listing to your shared drive)

What is the price of a 2,200 square foot house (+/- 100 square feet).....\$ _____
(Save a PDF one page copy of the real estate listing to your shared drive)

Qualifying for a Mortgage

Go to calculator.com and find the “How much house can I afford Calculator”

Enter the following information:

Average annual USA income	From Above
Monthly debt	\$750
Down payment	\$5,000
Property Tax Rate	1%
Home Insurance Rate	0.5%
Interest Rate	8.00%
Length of Loan	30 years

Can you afford either the 1,400 or 2,200 square foot house in the neighborhood you selected? If not, find a less expensive house that qualifies for a loan or find new houses in a less expensive neighborhood and repeat the steps above. If your chosen career pays less than the US average, what does this mean in terms of your future housing plans? How long will you have to live somewhere else before you can save up the \$5,000 for a down payment? Where will you live?

Mortgage Calculation and Amortization

Go to calculator.com and find the “Mortgage Payment Calculator”

Enter the following information for the 1,400 square foot house:

Mortgage Amount	House Price - \$5,000
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Interest Rate	6.0%
Amortization Length	15
Starting Month	Next Month, This Year
Prepayment Method	None
Prepayment Amount	None
Prepayment after Month	None
Full Amortization Table	Yes
Display Using	HTML 3.0 Tables

What is the monthly mortgage payment?.....\$ _____

(Save a PDF copy of the amortization table to your shared drive)

Enter the following information for the 2,200 square foot house:

Mortgage Amount	House Price - \$5,000
Interest Rate	6.0%
Amortization Length	30
Starting Month	Next Month, This Year
Prepayment Method	None
Prepayment Amount	None
Prepayment after Month	None
Full Amortization Table	Yes
Display Using	HTML 3.0 Tables

What is the monthly mortgage payment?.....\$ _____

(Save a PDF copy of the amortization table to your shared drive)

What is the main difference between these two calculations?

What was the monthly payment difference between these two houses?.....\$ _____

Which house would you buy and why?

Which loan would you choose and why?

Mortgage Retirement Strategy

Calculate the amount you would have for retirement if you invested the monthly payment difference at a rate of return of 5% for 15 years.....\$ _____

Calculate the amount you would have for retirement if you invested the full 15 year mortgage payment at a rate of return of 5% for 15 years.....\$ _____

If you add these two amounts to the amount you would have for retirement if you invested the 15% at a rate of return of 5% for 30 years.....\$_____

Does the math your parents used to achieve their retirement and buy their life back still hold true for you? If not, is there a strategy you can use to retire at the same point your parents did, even if the stock market pays less than half what they paid your parents? Are you in charge of buying your life back, or can you leave that decision to someone else? Why or why not?

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Student handout and computer internet access

Set-Up Required:

- Computer lab student access

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

- Students can connect the logic developed in this exercise to begin work on a Business Financial Plan, Personal Finance, or Securities & Investments and compete within FBLA guidelines
- FBLA provides leadership activities that correlate with the Mathematics curriculum. At a minimum:
 - 1.1 The student will analyze, refine, and apply decision-making skills through classroom, family, community, and business and industry (work-related) experiences.
 - 21st Century interdisciplinary theme--financial, economic, business & entrepreneurial literacy: Critical Thinking and Problem Solving
 - Solve Problems

Cooperative Learning:

- Students who complete each section will assist those students who have not finished yet

Expectations:

- All students will complete all sections and save their work to their shared drive as a PDF file

Timeline:

- 2 50 minute class periods

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- Students can better understand the relationship between employment and income and lifestyle

Career Applications

- Banking and Finance

Optional or Extension Activities

- Research Fidelity Mutual Funds and develop an investment strategy that meets or risk/return model and target rate of return
- Use the cash flow spreadsheet to look at monthly expenses and determine what you would be willing to have less of to qualify for a more expensive house

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FUTURE VALUE - PERIODIC INVESTMENT

	B	Ending Balance
400	p	Monthly Retirement Contribution
0.032	r	Interest rate expressed as a decimal
12	n	Number of times interest is compounded annually
28	t	Number of years

\$217,029.69

Ending Balance

Applied

Math

217030

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P	Par en thesis
E	Ex po nent
M	M u ltiplication
D	Di vi sion
A	Ad di tion
S	Sub tr action

Washington

Applied

Math

Council

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Category	4 - Exceeds Standard	3 - Meets Standard
Calculations	Calculations are completed, saved as a PDF on the shared drive, and explained or used in a referential context	Calculations are completed, and saved as a PDF on the shared drive
Written Response	Answers all items contained in the request, then explains the relevance of the answer, or links to previous or subsequent questions	Answers all items contained in the request
Logic & Reasoning	Response is clear, coherent, sequential, and based upon evidence or experience and then explains the relevance of the response, or links to previous or subsequent responses	Response is clear, coherent, sequential, and based upon evidence or experience
Timeliness	Assignment was complete and submitted before the due date	Assignment was complete and submitted on the due date

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2 - Approaches Standard	1 - Does Not Meet Standard
Calculations are completed but not saved as a PDF on the hard drive	Calculations are not completed
Answers less than all items contained in the request	Does not answer the request
Response is unclear but sequential, and based upon evidence or experience	Response is unclear, not sequential, or not based upon evidence or experience
Assignment was complete and submitted after the due date	Assignment was not completed and submitted

WAMC Lesson Plan

Name(s): Claus Joens
Email Address: cjoens@concrete.k12.wa.us
Lesson Title: Pensions
Date: 25 June 2019
Text: FA 2nd Ed STEM Correlation: Mathematics
Lesson Length: 3 Class Periods (+ 2 Class Period Lab)

Big Idea (Cluster): Economics & Finance
Planning for Retirement

- Retirement Income from Savings
- Social Security Benefits
- Pensions
- Life Insurance

Mathematics K–12 Learning Standards:

Communications

COMMON CORE

Speaking and Listening Standards

SL4 Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.

SL5 Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.

SL6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Educational Technology

1.3.2 Locate and organize information from a variety of sources and media.

1.3.3 Analyze, synthesize and ethically use information to develop a solution, make informed decisions and report results.

2.1.1 Practice personal safety.

2.1.2 Practice ethical and respectful behavior.

2.4.1 Formulate and synthesize new knowledge.

Math

COMMON CORE

A-SSE Interpret the structure of expressions.

A-CED Create the equations that describe numbers of relationships

F-IF Analyze functions using different representations

F-BF Build a function that models a relationship between two quantities

S-MD Calculate expected values and use them to solve problems

Reading

COMMON CORE

RST7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

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Health and Fitness

3.2.1 Evaluates health and fitness information, products and services.

Language

COMMON CORE

L4 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 9–10 reading and content, choosing flexibly from a range of strategies.

EMBEDDED LEADERSHIP ACTIVITIES

FBLA provides leadership activities that correlate with the Mathematics curriculum. At a minimum:

1.1 The student will analyze, refine, and apply decision-making skills through classroom, family, community, and business and industry (work-related) experiences.

21st Century interdisciplinary theme--financial, economic, business & entrepreneurial literacy: Critical Thinking and Problem Solving

Solve Problems

21st Century Skills

2.D.2 Identify and ask significant questions that clarify various points of view and lead to better solutions

Insurance comparison project.

PS 6: Identify different types of risks and how to protect against them.

Math Practice 3—Construct viable argument and critique reasoning of others.

Math Practice 4—Model with mathematics (apply mathematics to solve problems arising in everyday life, society and the workplace)

Math Practice 5—Use appropriate tools and estimate strategically.

Math Practice 6—Attend to Precision

Math Practice 8--Look for and express regularity in repeated reasoning

Mathematical Practice(s): multiplication, linear functions, exponential functions, Microsoft Excel spreadsheets

Content Objectives: Understand the types of pensions, pension guidelines, pension objectives, competing priorities, changing regulations

Language Objectives (ELL): Understand the common language of investment and retirement.

Vocabulary: deferred compensation, pension, defined benefit plan, vested, single life annuity, qualified joint and survivor annuity, lump-sum payment, Pension Benefit Guaranty Corporation, Employee Retirement Income Security Act of 1974 (ERISA), Pension Protection Act, cost of living adjustment, Consumer Price Index, Kline-Miller

Connections to Prior Learning:
2-3 Interest Rates, 2-4, 2-5 Compound Interest, 2-7 Future Value of Investments, 3-2 Loans, 3-4 Loan Calculations and Regression,

Questions to Develop Mathematical

Thinking:

- How much will I need?
- How long will it take?
- What happens if I change my mind?
- What are the variables?
 - term
 - rate

Common Misconceptions:

- Retirement is provided for you
- Everyone gets the same retirement
- It is something you decide on later in life

WAMC Lesson Plan

- | | |
|--|--|
| - compounding
• How will I reach my goal? | |
|--|--|

Assessment (Formative and Summative):

- | |
|---|
| • The student applies understanding of mathematic concepts and systems to analyze the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations in real world situations. |
|---|

Materials:

- | |
|--|
| • Internet search of pension goal requirements and historical returns 1929-2008; create an Excel spreadsheet to calculate retirement amounts and graph the results |
|--|

Instruction Plan:

Introduction: Day 1 Vocabulary and Concepts; Day 2 Excel Spreadsheets; Day 3 Application Problems

Explore: Major financial investments and the impact on your standard of living; intent vs impact of mortgage vs. retirement; regulatory instability

When I observe students: Students will work together in pairs with one above average student paired with one below average student
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Questions to Develop Mathematical Thinking as you observe: Do you need to memorize the formula or the concept? Is a formula a shortcut or creating incremental work? How will I know if I am using the correct formula? What are the characteristics of a linear vs. Exponential equation?
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Answers: Memorize concepts. You can always Google formulas. A formula is always a shortcut so it is important to understand what a formula is actually doing. Check your result against your expectation. If the result is continuous over time graph it and make sure the result is consistent. Linear equations do not change inputs or results over time (nothing changes). Exponential equations change inputs or results over time (something changes).
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Summarize: Retirement is buying your life back. You are responsible for your own retirement. The environment is constantly changing. Make your life goals and stick to them. Start saving right away. Estimate your retirement amounts and check them annually. Forecast at what age when you think you will retire.
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Career Application(s):

- | |
|--|
| • All careers with employee withholding and benefit packages |
|--|

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WAMC Lesson Plan

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)			
<input checked="" type="checkbox"/> Global Awareness	<input checked="" type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy	<input checked="" type="checkbox"/> Civic Literacy	
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21st Century Skills (Check those that students will demonstrate in the above activity.)			
LEARNING AND INNOVATION <u>Creativity and Innovation</u> <input checked="" type="checkbox"/> Think Creatively <input checked="" type="checkbox"/> Work Creatively with Others <input checked="" type="checkbox"/> Implement Innovations <u>Critical Thinking and Problem Solving</u> <input checked="" type="checkbox"/> Reason Effectively <input checked="" type="checkbox"/> Use Systems Thinking <input checked="" type="checkbox"/> Make Judgments and Decisions <input checked="" type="checkbox"/> Solve Problems <u>Communication and Collaboration</u> <input checked="" type="checkbox"/> Communicate Clearly <input checked="" type="checkbox"/> Collaborate with Others	INFORMATION, MEDIA & TECHNOLOGY SKILLS <u>Information Literacy</u> <input checked="" type="checkbox"/> Access and Evaluate Information <input checked="" type="checkbox"/> Use and manage Information <u>Media Literacy</u> <input checked="" type="checkbox"/> Analyze Media <input checked="" type="checkbox"/> Create Media Products <u>Information, Communications and Technology (ICT Literacy)</u> <input checked="" type="checkbox"/> Apply Technology Effectively	LIFE & CAREER SKILLS <u>Flexibility and Adaptability</u> <input checked="" type="checkbox"/> Adapt to Change <input checked="" type="checkbox"/> Be Flexible <u>Initiative and Self-Direction</u> <input checked="" type="checkbox"/> Manage Goals and Time <input type="checkbox"/> Work Independently <input type="checkbox"/> Be Self-Directed Learners <u>Social and Cross-Cultural</u> <input checked="" type="checkbox"/> Interact Effectively with Others <input checked="" type="checkbox"/> Work Effectively in Diverse Teams	<u>Productivity and Accountability</u> <input checked="" type="checkbox"/> Manage Projects <input checked="" type="checkbox"/> Produce Results <u>Leadership and Responsibility</u> <input checked="" type="checkbox"/> Guide and Lead Others <input checked="" type="checkbox"/> Be Responsible to Others

Embedded Leadership Activities

FBLA provides leadership activities that correlate with the Mathematics curriculum. At a minimum:

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Math Practice 8--Look for and express regularity in repeated reasoning

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Financial Algebra Quiz

Chapter 10, Section 3 - Pensions

1.) What is the average minimum amount you must save up to retire?

Answer: \$750,000 Answer is given in class

2.) In 1980, 60 % of workers were covered under a defined pension plan. Today the number is 4 %.

3.) Alex is 35 years old. He plans to retire when he is 63. He has opened a retirement account that pays 3.2% interest compounded monthly. If he makes monthly deposits of \$400, how much will he have in the account when he retires?

Answer: \$217,029.69 (From 10.3 MS Excel Worksheet)
(Students use MS Excel Worksheet with different numbers on daily work and the instructor changes the numbers for the quiz)

4.) Amanda is retiring after 30 years with a local manufacturing company. The company offers her \$60 for each year she has worked for the company. How much will her monthly pension be?

Answer: \$1,800
 $30 \times \$60 = \$1,800$

5.) Integrated Technologies offers employees a flat pension plan in which a predetermined dollar amount (multiplier) is multiplied by the number of years of service to determine the monthly pension benefit using the schedule shown. After working at Integrated Technologies for 22 years, Al decided to retire. He has been told there will be a 2.2% cost of living adjustment soon after he retires. Which will yield a higher retirement benefit, calculating the COLA on the multiplier or calculating the COLA on the monthly benefit using the following chart

Years Employed	Multiplier
15-19	\$40
20-25	\$45
30+	\$55

(You must show your work and explain your answer. As with all critical thinking, it must be logical, sequential, and based upon evidence or experience)

Answer: They are exactly the same
 $22 \times \$45 = \$990 \times 0.022 = \$21.78 + \$990 = \$1,011.78$
 $22 \times (\$45 \times 1.022) = 22 \times \$45.99 = \$1,011.78$