

# WAMC Lesson Plan

Name(s): Kari L. Toms, ktoms@eagles.edu

Lesson Title: Present Value of Investments

Date:

Text: Financial Algebra, 2<sup>nd</sup> Edition, Gerver/Srgoi

Lesson Length: 3 period (55 min)

Domain: Interpreting Functions

Big Idea (Cluster): Interpret Functions that arise in applications in terms of context

Common Core State Standards: F-IF4, A-SSE3, A-CED4, F-IF8b

Mathematical Practice(s): Functions, graphing

Content Objectives: Calculate PV

Language Objectives:

Vocabulary: Periodic Investment, Rates

Connections Prior to Learning

Questions to Develop Mathematical Thinking:

Common Misconceptions:

- |   |   |
|---|---|
| <ul style="list-style-type: none"> <li>•</li> </ul> | <ul style="list-style-type: none"> <li>• Present Value dollars are always worth more (Cash is King) In Economics, maybe, but PV is powerful goal-setting method of compound forecasting.</li> </ul> |
|---|---|

Assessment (Formative and Summative):

- |  |
|--|
| <ul style="list-style-type: none"> <li>• Formative: vocabulary worksheet comprehension and equation graphing; daily check-out</li> <li>• Summative: Quiz after both single deposit and periodic deposit investment, showing both formulas, substitution, and graphed solutions.</li> </ul> |
|--|

Materials:

- |  |
|--|
| <ul style="list-style-type: none"> <li>• 3-8 Worksheet 1</li> <li>• 3-8 Worksheet 2 – “Saving for a car” with Mini-Lab</li> <li>• MS Excel</li> <li>• Scientific Calculator</li> </ul> |
|--|

Instruction Plan: (Day 1)

Launch: Goal Setting Ideas discussion with students? What do you see a big ticket spending items in your future? How long from now? How much does that cost? Can we invest to get ahead of that?

Explore: Warm-up questions on board – then Worksheet 3-8 (1) on own.

*ASK: Does anyone have a big ticket item they're saving for? How much does that cost? Gather student responses on board. I.E. List items like: new motorcycle, \$6,500. Community College Tuition, \$3,000, New Tires, \$500, etc.*

*WRITE ON BOARD: Learning Target: I can calculate the present value dollars needed to be invested to reach a goal.*

GOALS ON BOARD

*Ask: Are you going to set aside money in one-lump sum or a little each month? Show hands.*

Single-deposit (Lump-sum) Formula

$$PV = \frac{FV}{(1 + r/n)^{nt}}$$

Motorcycle Example \$6,500 is needed in two years.

The local bank will pay 7.1% a year in a money market account, compounded monthly. Let's substitute values into the formula.

## WAMC Lesson Plan

$$PV = \frac{6,500}{(1.005916)^{24}}$$

The single-deposit needed to save for \$6,500 in the future is \$5641.90. How much interest will this person have earned?  $\$6500 - \$5641.90 = \$858.10$  will have been earned in "free money" by allowing interest to accrue.

What if you don't have \$5641 right now? You can attack this another way, by saving dollars each month at present value for use later on. This formula works like this:

$$P = \frac{FV \times (r/n)}{(1 + r/n)^{nt} - 1}$$

$$P = \frac{6,500 \times .005916}{(1 + .005916)^{24} - 1}$$

$$P = \frac{38.454}{.15208}$$

**P=\$252.85** You would need to save \$252.85 a month. That sounds more reasonable. If you're working at Washington minimum wage (9.32/hr in 2014), you'll take home about 85% of your earnings, which is 7.92/hr. How many hours a month would you need to work in order to make the deposit for the motorcycle? Round to the next full hour.  $252.85 / 7.92 = 31.93$ , 32 hours of work would be needed to make this savings commitment.

One more example before I turn them loose on their own:

New Tires for my car:

Stacia will need new snow tires next winter for her car. She has a quote for \$500 for the new tires. She'll need them in about 12 months. If she uses present value dollars to save and invest at 8% nominal interest, compounded monthly, how much will she need to deposit in a single deposit if she wants to buy the tires?

$$PV = \frac{FV}{(1 + r/n)^{nt}}$$

$$PV = \frac{500}{(1 + .00667)^{12}}$$

$$PV = \frac{500}{(1.00667)^{12}}$$

**P=\$461.66** Stacia would need to set aside 461.66 now to have 500 in 12 months.

## WAMC Lesson Plan

What would Stacia need to save each month, if she decided to make a deposit each month for the tires?

$$P = \frac{FV \times (r/n)}{(1 + r/n)^{nt} - 1}$$

$$P = \frac{500 \times (.00667)}{(1 + .00667)^{12} - 1}$$

$$P = \frac{3.335}{.08304}$$

**P=40.16.** Stacia would need to save 40.16 a month to get the new tires in 12 months. How many hours will she need to work to set aside this much? Round to the next whole hour. She earns 9.32/hr and takes home about 85% of her pay.  $9.32 \times .85 = 7.92/\text{hr}$   $40.16/7.92 = 5.07$  hours or 6 hours a month just to save for tires.

Think: How much total did Stacia earn in interest by saving month at a time?

STRATEGY:

12 months \* 40.16 per month = 481.92 in payments

\$500 value – 481.92 = 18.08 in interest earned

Now, hand out the worksheets to have students solve items on their own.

Allow 15-18 minutes for students to calculate data. At the conclusion (ticket out), students will have their progress documented by the teacher and recorded on the class roster/gradebook.

DAY 2-

Students will calculate their present value functions to achieve real goals.

*Write on board: I can calculate and graph the present value function, using coordinate pairs.*

*Saving for a car handout is distributed to students. Three car options are listed. Students will need to calculate the monthly periodic investment amounts needed to purchase a car in 2 years, record the data and reason if that would be possible on those wages.*

For those needing more time, modify the assignment to only compare the 2/4<sup>th</sup> (even) options or odd options.

For extended learners, have them either:

- find another job and re-adjust the values with another income level. Try [Go2worksource.com](http://Go2worksource.com) or other job posting sites. Print out job posting and attach to worksheet.
- Find another automobile of their own choosing and calculate the deposit and work hours necessary to commit that present value to investing.

OFFER QUIZ AT BEGINNING OF DAY 3

Review answers to quiz at end of day 3.

When I observe students: Verify formatively that they are de-constructing problems, underlining essential items and restructuring items into formula properly. Make sure they are using parentheses in the calculator!

# WAMC Lesson Plan

Questions to Develop Mathematical Thinking as you observe:

- A) Is it better to make many small deposits or one large one?  
 B) How can I compare interest rates, when they compound at varied frequencies?

Answers:

- A) Answers vary, but should consider the number of compounds, as well as the rate for a factor calculation.  
 B) Show them how a factor is derived to find what-if scenarios for comparison.

Summarize: Can students see the effect of saving early in small amounts for investments? Discuss the growth. For new learners, the project may be initially scaffolded with table factors pre-determined to demonstrate the principle with a simpler approach. Administer Quiz at beginning of Day 2, prior to discussing next future value concept.

Career Application(s):

- Investing percentages of income earned into investment accounts
- Saving income for goals determined (car, tuition, travel, etc)

21<sup>st</sup> Century Skills and Interdisciplinary Themes:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

- Global Awareness       Financial/Economic/Business/Entrepreneurial Literacy       Civic Literacy  
 Health/Safety Literacy       Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

**LEARNING AND INNOVATION**

Creativity and Innovation

- Think Creatively  
 Work Creatively with Others  
 Implement Innovations

Critical Thinking and Problem Solving

- Reason Effectively  
 Use Systems Thinking  
 Make Judgments and Decisions  
 Solve Problems

Communication and Collaboration

- Communicate Clearly  
 Collaborate with Others

**INFORMATION, MEDIA & TECHNOLOGY SKILLS**

Information Literacy

- Access and Evaluate Information

- Use and manage Information

Media Literacy

- Analyze Media  
 Create Media Products  
Information, Communications and Technology (ICT Literacy)  
 Apply Technology Effectively

**LIFE & CAREER SKILLS**

Flexibility and Adaptability

- Adapt to Change  
 Be Flexible

Initiative and Self-Direction

- Manage Goals and Time  
 Work Independently  
 Be Self-Directed Learners

Social and Cross-Cultural

- Interact Effectively with Others  
 Work Effectively in Diverse Teams

**Productivity and Accountability**

- Manage Projects  
 Produce Results  
Leadership and Responsibility  
 Guide and Lead Others  
 Be Responsible to Others

Scaffolding for newer learners: Use table factors for periods.

Extension for advanced learners: Utilize amortization and sinking fund problems for present and future values over time. Graph account values in sinking funds and amortization.

Chapter 3-8 Problems

1. In reference David Bach quote, students should be reflecting upon financial goals before they can calculate what they need. For example, saving tuition for college, money for a car, cannot begin with any accuracy until a goal figure is determined.

2.

- a. 889.00
- b. 2154.17
- c. 6084.13
- d. 40,136.04

3.

- a. 5285.49
- b. 3043.89
- c. 2072.04
- d. 2726.47

4.

$$P = \frac{50,000 \times (.042)}{(1.042)^7 - 1}$$

*Periodic Deposit*

$$P = \frac{2,100}{.33375}$$

*Periodic Deposit*

6292.13

5. 182.53

$$P = \frac{80,000 \times (.00229)}{(1.00229)^{240} - 1}$$

*Periodic Deposit*

$$P = \frac{183.20}{1.00369}$$

*Periodic Deposit*

6. 1651.89

$$P = \frac{10,000 \times (.00354)}{(1.00354)^6 - 1}$$

*Periodic Deposit*

$$P = \frac{35.40}{.02143}$$

*Periodic Deposit*

7. 260.34

$$P = \frac{10,000 \times (.00367)}{(1.00367)^{36} - 1}$$

*Periodic Deposit*

$$P = \frac{36.70}{.14097}$$

*Periodic Deposit*

8. 11,671.75

$$\text{Periodic Deposit } P = \frac{100,000 \times (.0195)}{(1.0195)^8 - 1}$$

$$\text{Periodic Deposit } P = \frac{1,950}{.16707}$$

9. 37,123.52

$$PV = \frac{50,000}{(1.015)^{20}}$$

10.

$$\text{Periodic Deposit } P = \frac{75,000 \times (.002583)}{(1.002583)^x - 1}$$

$$\text{Periodic Deposit } P = \frac{193.725}{(1.002583)^x - 1}$$

NAME: \_\_\_\_\_

PERIOD: \_\_\_\_\_

### Chapter 3-8 Worksheet 1: Calculating Present Value of an Investment

#### PV OF A SINGLE DEPOSIT

The single deposit formula can be used to find the present value of a one-time investment.

$$PV = \frac{FV}{(1 + r/n)^{nt}}$$

1. Underline the elements needed in this problem to fulfill the formula listed above, then calculate the amount needed:
  - a. Shyandra would like to be able to purchase a car after graduation. She thinks she will need about \$4,000 for her first car. How much will she need to deposit into an account now to have the car in 18 months? Her credit union is currently paying 5% on 6-month, renewable CDs.
  - b. Would it be a better deal for Shyandra to not bank at her usual credit union and place her car money into an account that yielded 8%, compounded quarterly for the next year-and-a-half?

#### PV of Periodic Deposit Investments

The periodic deposit formula can be used to find the present value of a repeating investment.

$$P = \frac{FV \times (r/n)}{(1 + r/n)^{nt} - 1}$$

2. Underline the elements needed in this problem to fulfill the formula listed above, then solve for the present value needed.
  - a. Emmanuel would like to be able to pay for his community college tuition to graduate with his associate's degree debt-free. He estimates he will need about \$6,750 in two years. How much must he deposit each month, if his account yields 4.75%, compounded monthly?
  - b. How many hours each month must Emmanuel work just to earn this college money, if he works for \$9/hour gross, and his take-home wages are about 85% of the gross pay? Round up to the next full hour.

NAME: \_\_\_\_\_

PERIOD: \_\_\_\_\_

### Chapter 3-8 Worksheet 1: Calculating Present Value of an Investment

#### PV OF A SINGLE DEPOSIT

The single deposit formula can be used to find the present value of a one-time investment.

$$PV = \frac{FV}{(1 + r/n)^{nt}}$$

1. Underline the elements needed in this problem to fulfill the formula listed above, then calculate the amount needed:
  - a. Shyandra would like to be able to purchase a car after graduation. She thinks she will need about \$4,000 for her first car. How much will she need to deposit into an account now to have the car in 18 months? Her credit union is currently paying 5% on 6-month, renewable CDs. **3714.40**
  - b. Would it be a better deal for Shyandra to not bank at her usual credit union and place her car money into an account that yielded 8%, compounded quarterly for the next year-and-a-half? **Yes, she'd have to deposit less. 3551.89, a savings of 161.39**

#### PV of Periodic Deposit Investments

The periodic deposit formula can be used to find the present value of a repeating investment.

$$P = \frac{FV \times (r/n)}{(1 + r/n)^{nt} - 1}$$

2. Underline the elements needed in this problem to fulfill the formula listed above, then solve for the present value needed.
  - a. Emmanuel would like to be able to pay for his community college tuition to graduate with his associate's degree debt-free. He estimates he will need about \$6,750 in two years. How much must he deposit each month, if his account yields 4.75%, compounded monthly? **\$268.65**
  - b. How many hours each month must Emmanuel work just to earn this college money, if he works for \$9/hour gross, and his take-home wages are about 85% of the gross pay? Round up to the next full hour.
    - i.  $9 \times .85 = 7.65/\text{hr net pay}$
    - ii.  $268.65/7.65 = 35.11$ , rounds up to 36 hours a month

Name: \_\_\_\_\_  
 PERIOD: \_\_\_\_\_

**Chapter 3-8 Present Values,  
 “Shopping for a Car”**

Suppose you have landed a job, earning \$9.32/hr for the time being. If all goes well, after 6 months, you should receive an agreed upon raise to \$10.15/hr. Since you have a job, your parents have agreed to let you continue driving their car until you graduate. Now, you have to decide if you’d like to purchase a car after graduation. The four cars listed below are representative options of what is currently available in the marketplace.

Calculate the periodic deposit required of you to save up enough money for a car after graduation. You have two years to save the money. Your bank is offering an investment at 11% APR, compounded monthly.

$$P = \frac{FV \times (r/n)}{(1 + r/n)^{nt} - 1}$$

- 1) Calculate the monthly deposit required to save up for each one of these autos, using the present value, periodic deposit formula.
- 2) Calculate how many hours you’ll need to work just to make enough for the auto deposit for the first six months AND the remaining 18 months (after your raise).
- 3) Do you think you’d be able to save enough with the periodic deposits to purchase a car with your investment (as opposed to having a payment later?)
- 4) For extra credit, find and print out an advertisement for a car that you’d personally prefer and calculate the deposit needed, as well as the hours necessary to fund that account.
- 5) After calculating, try it again with 3 years to save – does that make it seem less expensive?

	<b>Option 1</b>	<b>Option 2</b>	<b>Option 3</b>	<b>Option 4</b>
	2014 Honda Civic Coupe	2014 Chevrolet Silverado 1500 LT	1968 VW Beetle	1994 Toyota Camry
	\$18,190	\$29,410	\$7,000	\$4500
				
<b>DEPOSIT</b>				
<b>Hours @ Wage 1</b>				
<b>Hours @ Wage 2</b>				

## Chapter 3-8 Present Values, "Shopping for a Car"

Suppose you have landed a job, earning \$9.32/hr for the time being. If all goes well, after 6 months, you should receive an agreed upon raise to \$10.15/hr. Since you have a job, your parents have agreed to let you continue driving their car until you graduate. Now, you have to decide if you'd like to purchase a car after graduation. The four cars listed below are representative options of what is currently available in the marketplace.

Calculate the periodic deposit required of you to save up enough money for a car after graduation. You have two years to save the money. Your take-home pay is about 82% of our hourly pay. Your bank is offering an investment at 11% APR, compounded monthly.

$$P = \frac{FV \times (r/n)}{(1 + r/n)^{nt} - 1}$$

- 1) Calculate the monthly deposit required to save up for each one of these autos, using the present value, periodic deposit formula.
- 2) Calculate how many hours you'll need to work just to make enough for the auto deposit for the first six months AND the remaining 18 months (after your raise).
- 3) Do you think you'd be able to save enough with the periodic deposits to purchase a car with your investment (as opposed to having a payment later?)
- 4) For extra credit, find and print out an advertisement for a car that you'd personally prefer and calculate the deposit needed, as well as the hours necessary to fund that account.
- 5) Now, try just the monthly deposit again with 3 years to save – does that make it seem less expensive?

	<b>Option 1</b>	<b>Option 2</b>	<b>Option 3</b>	<b>Option 4</b>
	2014 Honda Civic Coupe	2014 Chevrolet Silverado 1500 LT	1968 VW Beetle	1994 Toyota Camry
	\$18, 190	\$29,410	\$7,000	\$4,500
				
<b>Save</b>	681.02 for 2 yrs	1101.09 for 2 years	262.07 for 2yrs	168.48 for 2 yrs
<b>Hours first 6 months</b>	681.02/7.64=89.1 90 hours/mo	144.12, 145 hours	34.3, 35 hours	22.05, 23 hours/mo
<b>Hours /month</b>	681.02/8.32=81.8 82 hours/mo	132.34, 133 hours	31.49, 32 hours	20.25, 21 hours/mo
<b>3yr</b>	428.75	693.22	165.00	106.07

$$2 \text{ year model } \quad P = \frac{FV \times (.00917)}{(1.00917)^{24} - 1}$$

$$3 \text{ year model } \quad P = \frac{FV \times (.00917)}{(1.00917)^{36} - 1}$$

$$P = \frac{FV \times (.00917)}{.24493}$$

$$P = \frac{FV \times (.00917)}{.38904}$$



NAME: \_\_\_\_\_  
PERIOD: \_\_\_\_\_

### Chapter 3-8 Quiz, Present Value

1. Calculate the both the lump-sum and periodic deposit required of you to save up enough money for a \$10,000 car after graduation. You have two years to save the money. Your bank is offering an investment at 8% APR, compounded quarterly. Show your equations and calculations.

$$\text{Lump Sum Deposit} \quad P = \frac{10,000}{(1.02)^8}$$

Lump Sum Needed = \$8,534.90

$$\text{Periodic Deposit} \quad P = \frac{10,000 \times (.02)}{(1.02)^8 - 1}$$

Periodic Deposit Needed = \$1165.09

2. Suppose you'd like to save to buy a home when you turn 30 years old. In 13 years, will you be able to buy a \$500,000 home on an investment with APR of 7.25%, compounded monthly? What would be the monthly deposit needed to fund this account? Is this reasonable? Why or why not?

$$\text{Periodic Deposit} \quad P = \frac{500,000 \times (.00604)}{(1.00604)^{156} - 1}$$

$$\text{Periodic Deposit} \quad P = \frac{3,020}{1.55846}$$

**Periodic Deposit Required \$1937.81**