WAMC Lab Template

Math Concept(s): Explore and Find Maximum Volume with given surface area

Source / Text: Illustrative Math Gr 8 Unit 5 Lesson 3 Functions and Volume

Developed by: Marilyn Filley E-Mail: mlfilley@seattleschools.org Date: 6/26/23

Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

After prereq skills (Quadratic lesson) students will do lab. They will construct open-topped boxes of different heights from 8.5x11 sheets of paper. Students will need to delegate box sizes to construct. Students should find a pattern and determine how to best graph the data – ideally taping boxes to the board as data points. Students should wonder if more data is needed to be certain of max volume. The lab gives context to a cubic function.
[More advanced students may realize there is a cubic function for this problem during the lab, but is not a part of the lab. It is covered in the Optimize Volume Lesson that follows this lab.]

Lab Plan

Lab Title: Which Box holds the most popcorn?

Prerequisite skills: Volume of a rectangular prism, independent/dependent variable, express quantities in terms of one variable, domain and range.

Lab objective: To understand visually how a cubic function and its graph relate to volume.

Standards: *(Note SPECIFIC relationship to Science, Technology, and/or Engineering)*

Mathematics K–12 Learning Standards:

- SA-CED.A.4

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

- SA-REI.D.10

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)

- SF-IF.B.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Standards for Mathematical Practice:

- MP2: Reason Abstractly and Quantitatively
- MP3: Construct viable arguments and critique the reasoning of others

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- Initiate and participate effectively in a range of collaborative discussions (one on-one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

K-12 Science Standards

- HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Technology

- . Innovative Designer - Students use a variety of technologies within a design process to identify and solve problems by creating new, useful or imaginative solutions (graph in desmos graphing calc)

Engineering

-

Leadership/21st Century Skills:

1.B Work Creatively with Others

Student Outcome: The student will demonstrate the ability to incorporate and utilize the principles of group dynamics in a variety of settings

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness
☐ Health/Safety Literacy
☐ **X** Financial/Economic/Business/Entrepreneurial Literacy
☐ Environmental Literacy
☐ Civic Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

Creativity and Innovation
☐ **X** Think Creatively
☐ Work Creatively with Others
☐ Implement Innovations

Critical Thinking and Problem Solving
☐ **X** Reason Effectively
☐ Use Systems Thinking
☐ Make Judgments and Decisions
☐ Solve Problems

Communication and Collaboration
☐ **X** Communicate Clearly
☐ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

Information Literacy
☐ Access and Evaluate Information
☐ Use and manage Information

Media Literacy
☐ Analyze Media
☐ Create Media Products

Information, Communications and Technology (ICT Literacy)
☐ **X** Apply Technology Effectively

LIFE & CAREER SKILLS

Flexibility and Adaptability
☐ Adapt to Change
☐ Be Flexible

Initiative and Self-Direction
☐ Manage Goals and Time
☐ Work Independently
☐ **X** Be Self-Directed Learners

Social and Cross-Cultural
☐ **X** Interact Effectively with Others
☐ **X** Work Effectively in Diverse Teams

Productivity and Accountability
☐ Manage Projects
☐ Produce Results

Leadership and Responsibility
☐ **X** Guide and Lead Others
☐ Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- 8.5 x 11 standard paper, ruler, tape
- Whiteboard/chalkboard to make large graph with well-spaced intervals (or do on pavement with chalk)
- Optional access to graphing tech if students move through experiment easily

Set-Up Required:

- Possibly bring popcorn

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

- We need to make several boxes from standard size paper to determine the dimensions that hold the most volume. Students will make an open topped box from a single sheet of paper. Encourage student leadership to delegate shapes that pairs or groups can make. Determine independent variable to help organize sizes.

Cooperative Learning:

- Students help each other fold in or cut out corners that comply to their size box. Tape corners together. Label box dimensions and volume. Height is created by corner cut.

Expectations:

- Students will share what they suspect is best volume. Students should suggest how to organize as a graph; teacher then guides as little as possible how to put volume equation in one variable. Is the class convinced we found the best shape/dimensions to optimize volume? Do we keep building boxes at smaller measurement intervals?

Timeline:

- Lower ability - 2 x 50 min class periods; one to make boxes and explore patterns; one to graph, find cubic equation, find most accurate maximum.
- Higher ability - 1 class period.

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab – move on to Volume Lesson that follows Lab

- Business – minimize packaging materials to hold maximum volume

Career Applications

- Working collaboratively to explore, build, organize, model and predict

Optional or Extension Activities

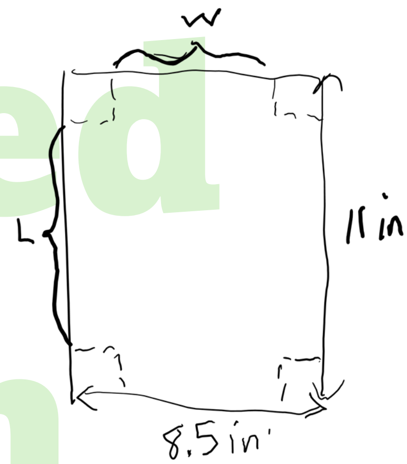
- Multiply out the trinomial to see another form of the equation. How the factored form informs the situation.

*Open in desktop app

Lab Instructions:

POPCORN BOX -
What shape box will hold
the MOST popcorn?

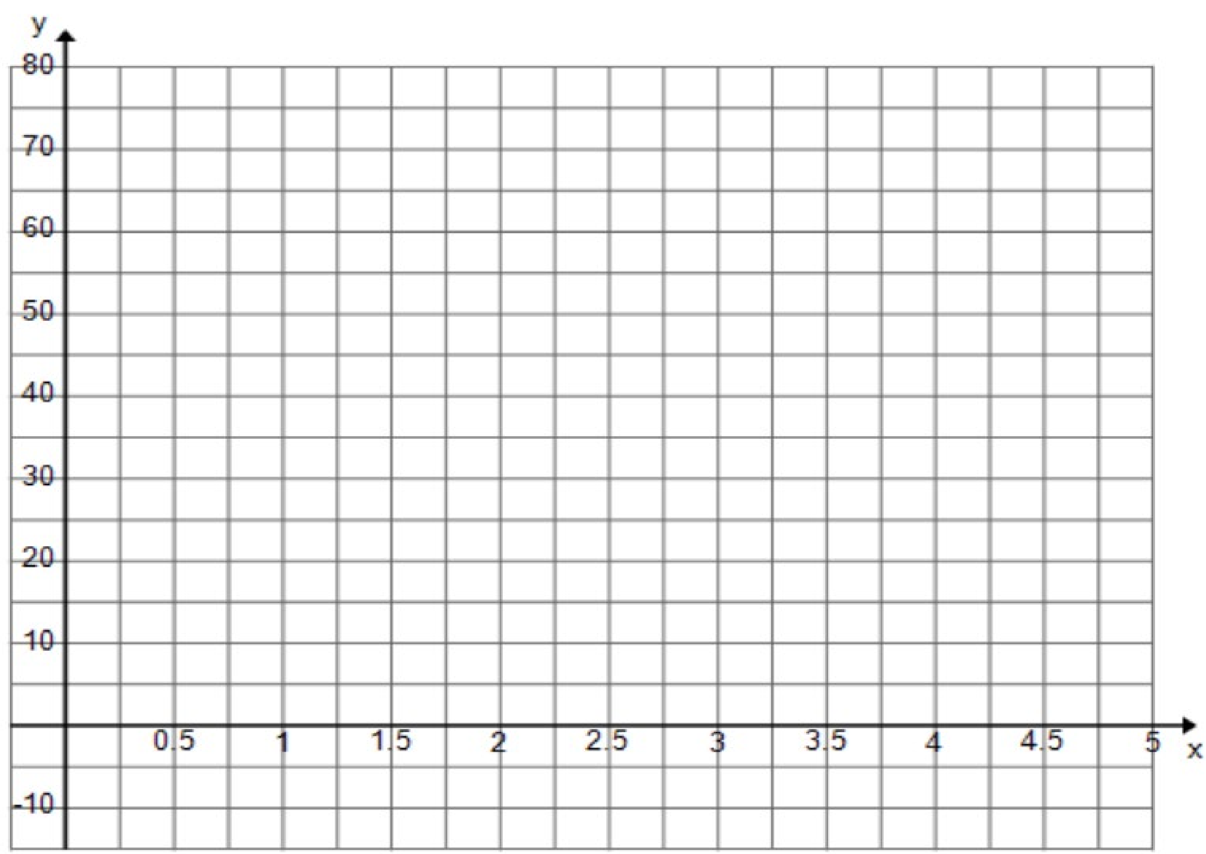
Use ONE 8.5 x 11 sheet of paper to make a popcorn box.
The top of the box is open. (Popcorn filled to rim)



This is a group effort – we need to use limited material
(a sheet of paper per box) to find maximum volume.

Groups or pairs?
Instructions for all on box construction? Inches or cm?

Student Handout: (Optional – Only after sorting out how to graph as a group)



Assessment Tool: Formative

- Can each pair or small group construct 1 – 3 boxes of different dimensions that do not duplicate other groups?
- Can the class/groups organize all boxes according to independent variable in some way to look for patterns and directional change from increasing to decreasing volumes?
- Can the class/group identify the need to move from discrete data to continuous data?
- Can pairs or groups work together to develop the cubic equation in terms of the independent variable?
- Can the class/group identify key point on the graph of the cubic equation (intercpepts, maximum, and why there are negative or infinite volumes that don't apply)?

See next page for Lab follow up: Optimize Volume Lesson – Open in desktop app

**OPEN DOCUMENT IN WORD DESKTOP APPLICATION

Name(s):Marilyn Filley
Email Address:mlfilley@seattleschools.org
Lesson Title: Optimize Volume Lesson
Date: 6/26/23
Text:Illust Math Geo Unit 2 /Gr8U5 STEM Correlation: Engineering Lesson Length: 50 min

Big Idea (Cluster): Explore box sizes to find max volume given limited surface area	
• Mathematics K–12 Learning Standards: SA-CED.A.4 Rearrange formulas, SA-REI.D.10 Understand graph as set of all solutions, SF-IF.B.4 Interpret key features of graphs	
Mathematical Practice(s): 2 – Reason abstractly and quantitatively 3 – Construct viable arguments and critique reasoning 6 – Attend to precision	
Content Objectives: Write, graph and interpret cubic functions representing volume	Language Objectives (ELL): Speaking and Listening,
Vocabulary: Local maximum, factors, roots, domain and range	Connections to Prior Learning Independent/Dependent Variable, Maximum value
Questions to Develop Mathematical Thinking: • What does the variable represent? • How do we write a similar equation with different parameters, different 3D shapes? • How do we attend to accuracy for the maximum in what contexts? • How do we read/interpret all features of the graph?	Common Misconceptions: • Assumptions about location of data points not tested (shape of curve) • Subtract height only once instead of twice from paper dimensions to arrive at box width and length • Other errors in writing cubic equation as product of width*length*height in one variable

- Assessment (Formative and Summative):

- Formative Assessment: Walk around and check for understanding after demonstrating a few requested problems if students need it.
- Summative Assessment:

1) Given a sheet of 11in x14in paper, write the dimensions (height, width, length) of one possible box that can be made in the same way as the lab. What is this box's volume?
2) Write the cubic equation for all volumes that can be created with the 11in x 14in paper. Express all dimensions in terms of one variable.
3) Challenge: Graph the above, adjusting axes. Find maximum volume on graph.

Materials:

- Worksheet

Instruction Plan: Explore how a cubic function provides most accurate dimensions for maximum volume.

Introduction: Let's continue the lab or use what we learned from the lab to make functions work for us – give us the most accurate information for our volume optimization.
Explore: How do we write a cubic equation for volume using our independent variable? We can work on this together.
When I observe students: Remind them how choosing height affected other measurements so that they graph needed variables. What do you need to do to be able to see the axis and the function clearly? Interpret/discuss graph.
Questions to Develop Mathematical Thinking as you observe: After discussing data, develop cubic equation: Refer students to drawing of 8.5 x 11 in sheet in Lab Instructions. Cutting corners created box height. This in turn affected box width and length. Once equation found, if you needed an equation for a different size paper, what stays the same and what changes?
Answers: Needed variables: Indep x= height, Dependent y = Volume. Students need to adjust window/axes and intervals to graph. Volume equation: V= h(8.5-2x)(11-2x) third degree. Students need to change constants in identical equation, graph, adjust to find new maximum for larger box.
Summarize: Can we explain how this graph relates to our boxes and optimizing volume?

Career Application(s):

- Packaging and Shipping

Leadership/21st Century Skills:

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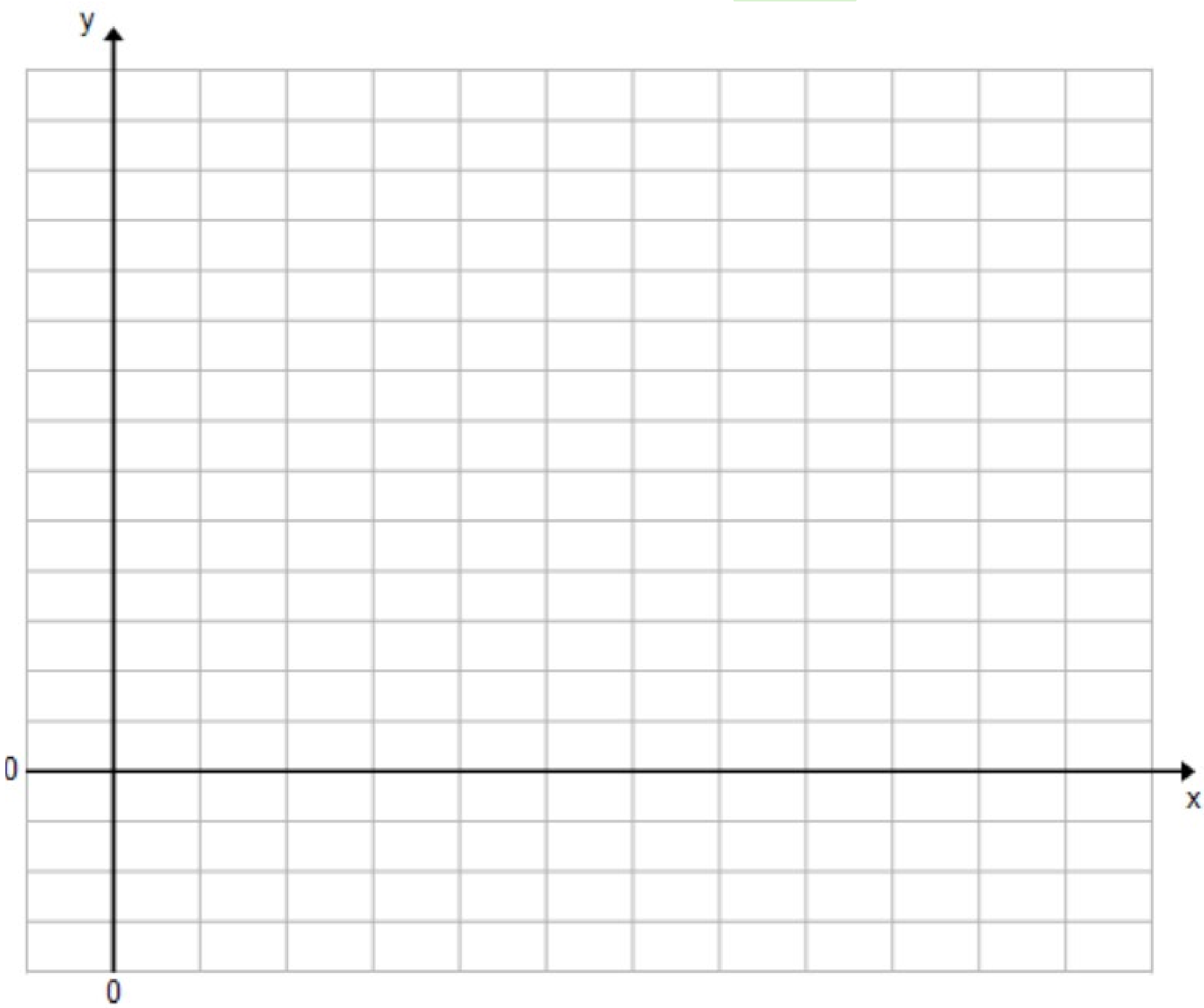
Optimize Volume Lesson – open in desktop app

1. Record the class data onto the graph Independent variable ____________________

Dependent Variable _____________________



2. What seems to be the height that makes the most volume? How certain are you?

3. What seems to be the shape of the graph? Is this a parabola? Why or why not?

4. What height would make zero volume? Is there more than one? Graph these points. using only independent variable height:

5. Rewrite Volume as a function using ONLY the independent variable:

____

6. Graph this in desmos or on graphing calc. Include it along with data on graph above.
Does the function give you a more accurate maximum volume? (_______ , _______)

7. Express with inequalities

DATA:
Domain ______________
Range _______________

FUNCTION:
Domain ______________
Range _________________

8. Why are the domain and range different? Did we miss negative and infinite volumes shown in the graph of the function?

9. You are exploring the best dimensions for a box made from a 30cm x 30cm piece of card board, following the same construction methods.
You plan to consider the volumes of the boxes made from this cardboard for every 2 cm change in height. Will you end up building the box with the maximum volume?
How could you convince someone else of your answer?

9. Challenge: What size circles and rectangle can be cut from a single 21cm x 28cm tin sheet to make a cylinder with the largest volume?

Volume of cylinder = ____________