WAMC Lab **Financial Algebra Chapter 3-6 Compound Interest**

Math Concept(s): Compound Interest and Exponents

Source / Text: Financial Algebra Southwestern Cengage Learning 2013, 2011

www.nsa.gov

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Date: Summer In-service 2013

Attach the following documents:

Lab Instructions:

Give each student Worksheet #1 and a piece of colored paper. Read and discuss the directions putting an emphasis on the questions that need to be answered. Students will fold the colored paper to figure out the exponential growth of rectangles when you fold.

Then give each student Worksheet #2 to help them think about how the Zombie Flu can spread exponentially.

Student Handout(s): Worksheet #1 Folding Paper and Worksheet #2 Zombie Flu

Rubric and/or Assessment Tool: Answer Key

Short Description (Be sure to include where in your instruction this lab takes place):

Students will explore exponential growth by looking at how the flu spreads in school and by folding paper.

They will discover the relationship between the number of folds on a piece of paper and the number of rectangles produced. They will fold rectangles over and over, procuring 'exponentially more' rectangles with each fold. Give each student Worksheet #1 and a piece of colored paper. Read and discuss the directions putting an emphasis on the questions that need to be answered.

Then give each student Worksheet #2 to help them think about how the Zombie Flu can spread exponentially.

Lab Plan

Lab Title: Understanding Compounding and the Flu.

Teacher Facilitation

Review functions previously studied such as linear and quadratic, by having students complete an exercise to recall the algebraic and geometric characteristics of these functions:

- Give an example of a linear relationship. What is the slope-intercept form for the equation of a line?
- Write the standard form of the quadratic equation. Graph an example and describe key features of the graph.

Prerequisite skills:

- Be able to identify the dependent and independent variable for a data set.
- Recognize patterns in a plotted area
- Describe linear and quadratic formula
- Understand the Compound Interest Formula $B = p\left(1 + \frac{r}{n}\right)^{nt}$

Lab objective:

- Identify exponential growth as a model of real-life situations
- Construct and make inferences from graphs of exponential functions

Standards:

CCSS-M:

Domain: The Real Number System N-RN1, N-RN2
Big Idea (Cluster): Extend the properties of exponents to rational exponents.
Common Core State Standards: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for the notation for radicals in terms of rational exponents.
Common Core State Standards: Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Domain: Seeing Structure in Expressions A-SSE1b, A-SSE3
Big Idea (Cluster): Interpret the structure of expressions.
Common Core State Standards: Interpret expressions that represent a quantity in terms of its content.* b. Interpret complicated expressions viewing one or more of their parts as a single entity.
Big Idea (Cluster): Write expressions in equivalent forms to solve problems.
Common Core State Standards: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
Domain: Seeing Structure in Expressions A-SSE3c
Big Idea (Cluster): Write expressions in equivalent forms to solve problems
Common Core State Standards: Use properties of exponents to transform expressions for exponential functions.
Domain: Interpreting Functions F-IF8b
Big Idea (Cluster): Analyze functions using different representations
Common Core State Standards: Use the properties of exponents to interpret expressions for exponential functions.

Standards for Mathematical Practice:

Mathematical Practice(s): • MP.1 Make sense of problems and persevere in solving problems. • MP.5 Use appropriate tools strategically. • MP 6 Attend to precision • MP.7 Look for and make use of structure.

Reading:

Writing:

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness ☒ Financial/Economic/Business/Entrepreneurial Literacy ☐ Civic Literacy
☐ Health/Safety Literacy ☐ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION
Creativity and Innovation
☐ Think Creatively
☒ Work Creatively with Others
☐ Implement Innovations
Critical Thinking and Problem Solving
☒ Reason Effectively
☒ Use Systems Thinking
☒ Make Judgments and Decisions
☒ Solve Problems
Communication and Collaboration
☒ Communicate Clearly
☒ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS
Information Literacy
☒ Access and Evaluate Information
☒ Use and manage Information
Media Literacy
☐ Analyze Media
☐ Create Media Products
Information, Communications and Technology (ICT Literacy)
☐ Apply Technology Effectively

LIFE & CAREER SKILLS
Flexibility and Adaptability
☐ Adapt to Change
☐ Be Flexible
Initiative and Self-Direction
☒ Manage Goals and Time
☐ Work Independently
☐ Be Self-Directed Learners
Social and Cross-Cultural
☒ Interact Effectively with Others
☒ Work Effectively in Diverse Teams

Productivity and Accountability
☐ Manage Projects
☒ Produce Results
Leadership and Responsibility
☐ Guide and Lead Others
☐ Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Colored Paper
- Worksheets
- Pen

Set-Up Required:

- Just have the worksheets and paper ready.

Lab Organization Strategies:

Grouping/Leadership/Presentation Opportunities:

- Students should work as partners

Cooperative Learning:

- The students will share their results after the experiment.

Expectations:

- All students should be active and results will be recorded.

Timeline: This should take one or two class periods, depending on your students' math level.

Post Lab Follow-Up/conclusions:

Discuss real world application of learning from lab

- Where else can we use this type of exponential growth?
- What if the machine could continue to fold the paper? How many rectangles would we have?
- What about the virus? Can the zombie virus continue to spread exponentially?

Career Applications

Banker	Venture Capitalist	Economist
Builder	Investor	Insurance Agents
Real Estate Agents	Lawyer	Doctor
Teacher	Laborer	

Optional or Extension Activities

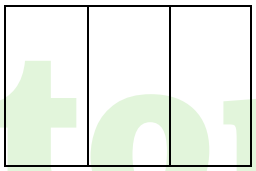
- Fold paper in half instead of 1/3.
- How long would it take to infect all of Washington with the Zombie Flu?
- Bring class together to share observations on the relationship between folding the paper and the number of rectangles formed.
- Focus on articulating a clear statement of an "exponential relationship."
- Have students graph their points (teacher produces graph on board/overhead).
- Teacher-directed class discussions on key features of the graph of the exponential function.
- Have students make comparisons between the exponential graph and linear/quadratic models.

Student Worksheet #1: Folding Paper

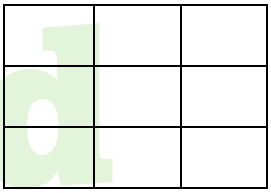
Name: _____ Date: _____ Period: _____

Directions:

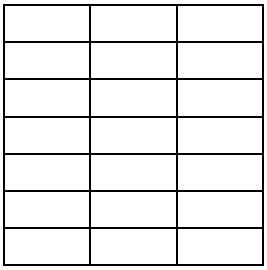
1) Fold a sheet of paper into 3 equal parts.
   Count how many rectangles were formed.



2) Fold the paper again into 3 equal parts.
   Count how many rectangles formed. Record your answer in the table.



3) Continue folding the paper into 3 equal parts, counting and recording the number of rectangles found until you can no longer make another fold.



Fold Stage	Number of Rectangles Formed
0	1
1	3
2	
3	
4	
5	
n	

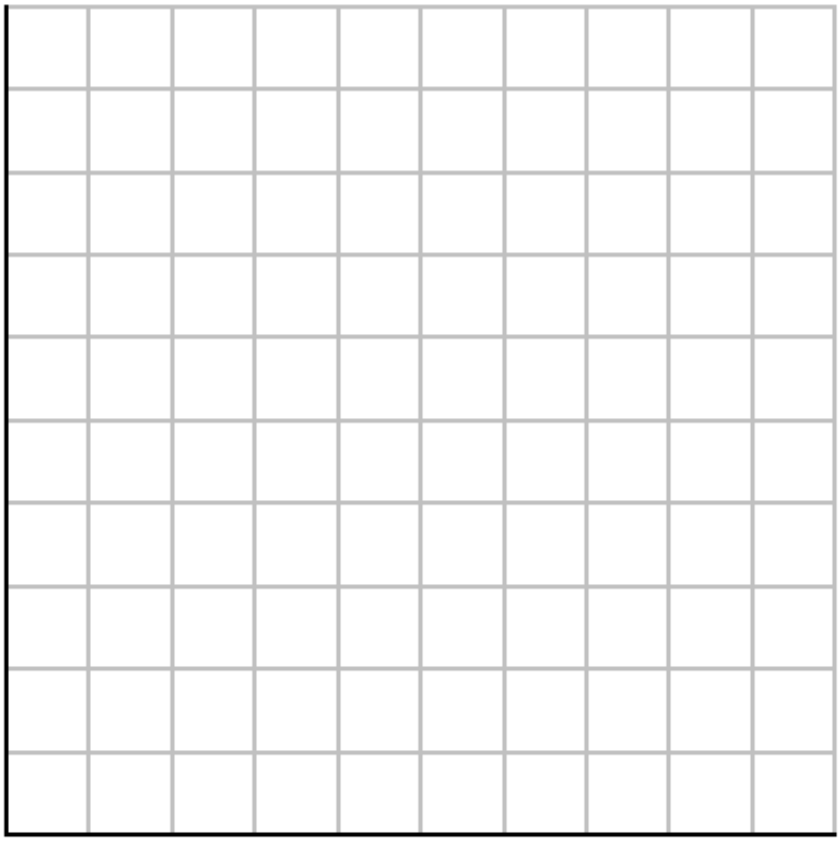
4) Examine the data gathered in the table. Did you see a pattern in the number of rectangles formed as the fold stage increases? Describe how the number of rectangles formed changes as the fold stages increase.

5) Write a rule to find the number of rectangles formed if the paper is folded *n* times.

6) Predict the number of rectangles after 10 folds.

7) If your paper now has 6561 rectangles, how many folds would you have made?

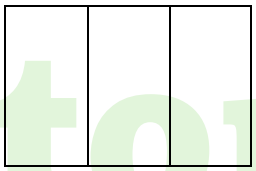
8) Make a sketch of the scatter plot on the grid below. Label the graph.



Student Worksheet #1: Folding Paper
Name: Answer Key Date: _____ Period: _____

Directions:

1) Fold a sheet of paper into 3 equal parts.
   Count how many rectangles were formed.



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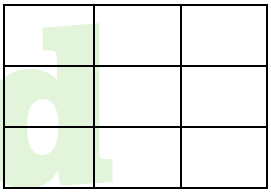

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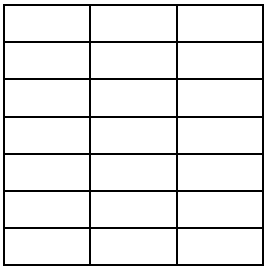


Fold Stage	Number of Rectangles Formed
0	1
1	3
2	9
3	27
4	81
5	243
6	729
7	2187
8	6561
n	3^n

4) Examine the data gathered in the table. Did you see a pattern in the number of rectangles formed as the fold stage increases? Describe how the number of rectangles formed changes as the fold stages increase.

Fold Number	**Number of Rectangles Formed**	**Folding Process**
0	1	
1	**3**	$3^1 = 3$
2	**9**	$3^2 = 3 \times 3$
3	*27*	$3^3 = 3 \times 3 \times 3$
4	*81*	$3^4 = 3 \times 3 \times 3 \times 3$
5	*243*	$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

5) Write a rule to find the number of rectangles formed if the paper is folded *n* times.

$$r = 3^n$$

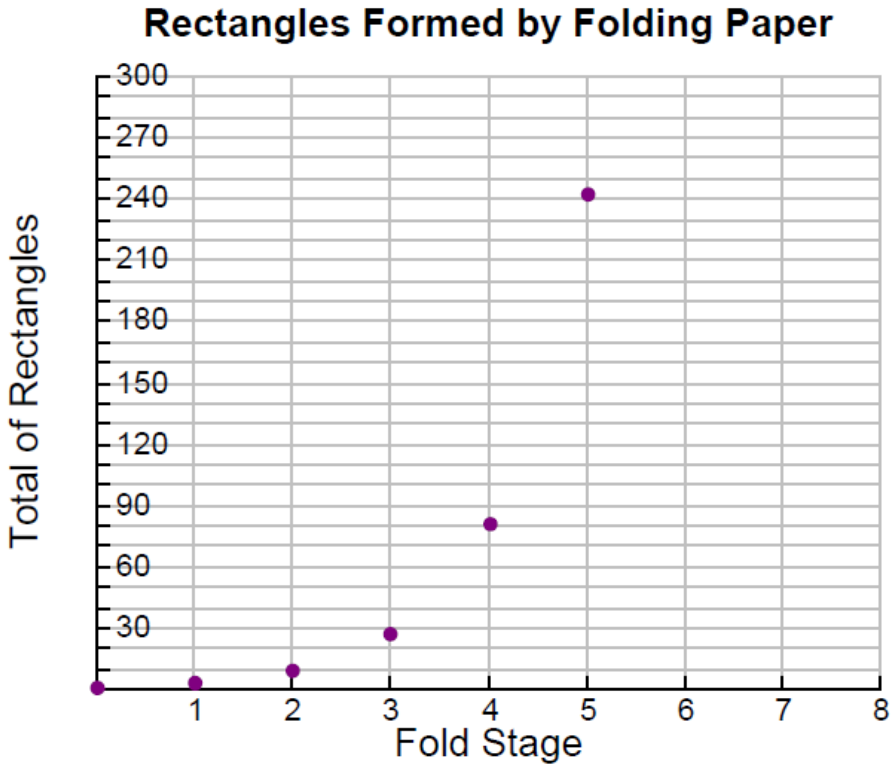
6) Predict the number of rectangles after 10 folds.

59049

7) If your paper now has 6561 rectangles, how many folds would you have made?

8 folds

8) Make a sketch of the scatter plot on the grid below. Label the graph.



Student Worksheet #2: Zombie Flu

Name: _____ Date: _____ Period: _____

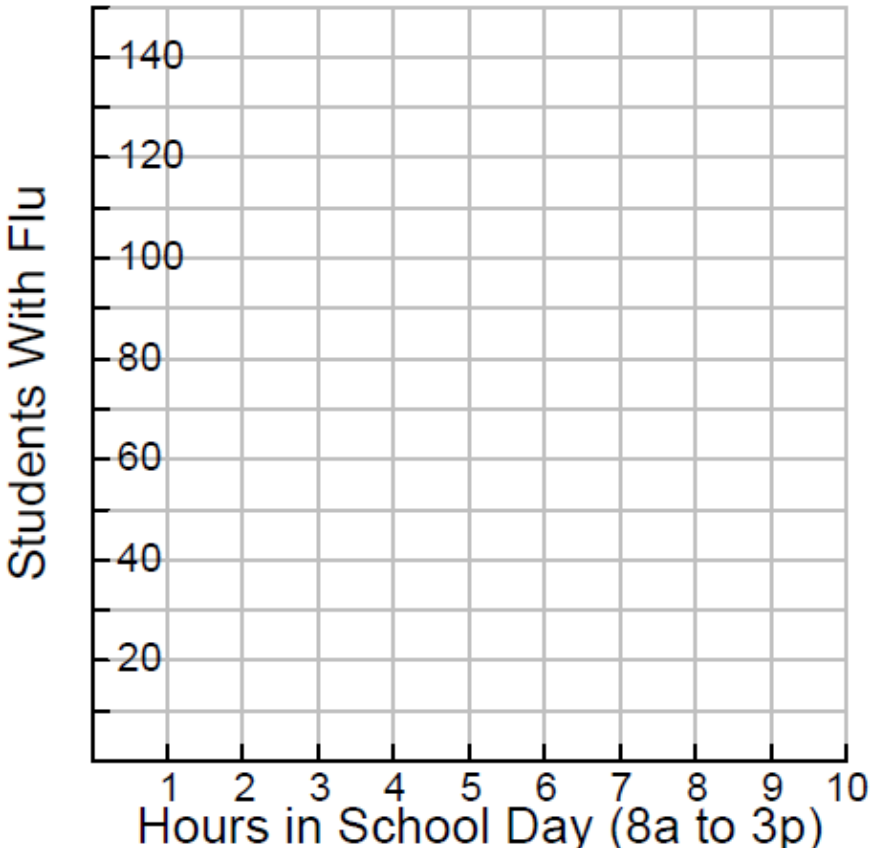
1) Elliot has the zombie flu, but he still comes to school on Monday. He arrives at 8am (*Hour 0*) and by 9am (*Hour 1*), Elliot has already bitten two of his friends, Brea and Taylor; they now have the zombie flu. By 10 am (*Hour 2*), Brea has bitten two of her friends, Sofia and Mike, and Taylor has bitten two of her friends, Raymond and Eduardo.

   a. If each person with the zombie flu bites two other people each hour, how many students are infected by 3pm (*Hour 7*)? Complete the table and graph the points in the grid below.

Table: *Number of Students Infected With the Zombie Flu by Hour*

	8am	9am	10am	11am	12pm	1pm	2pm	3pm
	Hour 0	**Hour 1**	**Hour 2**	**Hour 3**	**Hour 4**	**Hour 5**	**Hour 6**	**Hour 7**
# of new infections	1	2	4					
Total Infected	1	3	7					

Number of Students Infected With the Zombie Flu per Hour



b) If the school has 1,000 students enrolled, what percentage of the student body has the zombie flu by 3pm?

Student Worksheet #2: Zombie Flu
Name: Answer Sheet Date: _____ Period: _____

1) Elliot has the zombie flu, but he still comes to school on Monday. He arrives at 8am (*Hour 0*) and by 9am (*Hour 1*), Elliot has already bitten two of his friends, Brea and Taylor; they now have the zombie flu. By 10 am (*Hour 2*), Brea has bitten two of her friends, Sofia and Mike, and Taylor has bitten two of her friends, Raymond and Eduardo.
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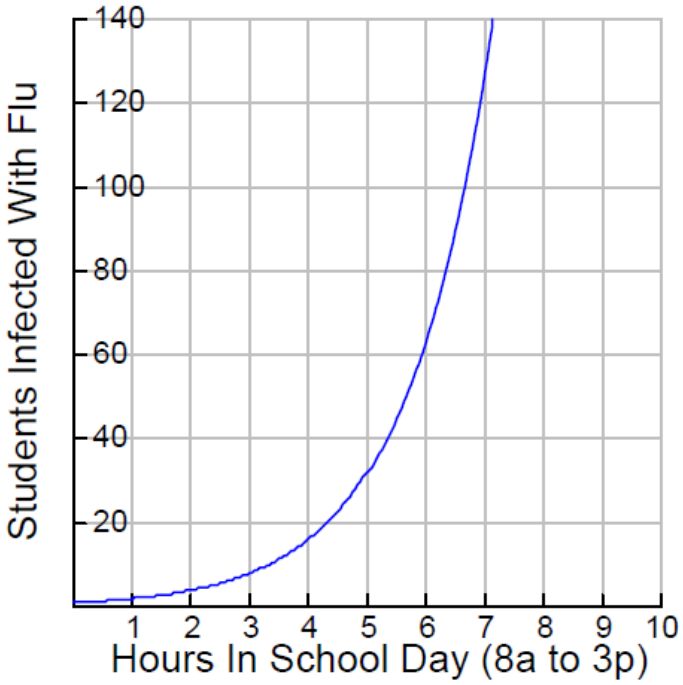
	8am	9am	10am	11am	12pm	1pm	2pm	3pm
	Hour 0	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6	Hour 7
# of new infections	1	2	4	8	16	32	64	128
Total infected	1	3	7	15	31	63	127	255

Number of Students Infected With the Zombie Flu per Hour



b) If the school has 1,000 students enrolled, what percentage of the student body has the zombie flu by 3pm?

25.5%

WAMC Lesson Plan

Name(s): Renee Crow
Lesson Title: 3-6 Continuous Compounding
Date: 6/26/13
Text: Financial Algebra Southwestern Cengage Learning 2013, 2011 **Lesson Length:** 2 days

Domain: The Real Number System N-RN1, N-RN2
Big Idea (Cluster): Extend the properties of exponents to rational exponents.
Common Core State Standards: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for the notation for radicals in terms of rational exponents.
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Big Idea (Cluster): Write expressions in equivalent forms to solve problems.
Common Core State Standards: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
Mathematical Practice(s): • MP.1 Make sense of problems and persevere in solving problems. • MP.5 Use appropriate tools strategically. • MP.6 Attend to precision.

Content Objectives: • I can use the Continuous Compounding Interest formula $B = pe^{rt}$ to solve problems	Language Objectives:
Vocabulary: **Continuous Compounding**: A method of calculating interest that is compounded an infinite number of times each year rather than being compounded every minute or even every microsecond. $B = pe^{rt}$ **Finite**: Something that has an end and can be represented by a real number. **Infinite**: Something without an end that cannot be represented by a real number. **Limit**: A concept in math that means an unreachable value.	**Connections Prior to Learning:** • In the last section, we were introduced to the compound interest formula. How could you use $B = p\left(1 + \frac{r}{n}\right)^{nt}$ to calculate something that compounds constantly?
Questions to Develop Mathematical Thinking: • Deep Thinking: How do you calculate infinity? What's the smallest measure of time? • Where can you look for information?	**Common Misconceptions:** • Order of Operations (PEMDAS) • How do I Factor? How do I separate the variable from the other numbers? • How do I revise the formula if I have different pieces of information? • How do I deal with an exponent when I'm factoring? How do I deal with an exponent when I'm separating the variable from the other numbers?

Assessment (Formative and Summative):

- Formative: watching students as they guess.
- Formative: listening to students talking in groups and sharing with class.
- Summative: Lab/Project
- Summative: Quiz

Materials:

- Books
- Laptops (Excel)
- Workbooks
- Tape
- Measuring stick

Instruction Plan:

Launch: If I gave you $1,000 at 100% interest compounded continuously for a year, how much would you have after the year? There are prizes for the table that gets the closest. *Take guesses from students.*

Explore: How could you solve this problem? *Possible Answers: compounded interest formula use* $B = p\left(1+\frac{r}{n}\right)^{nt}$ Students might determine the smallest amount of time for *n.* Students will work with table and then share ideas.
How much will you have if the $1,000 deposit compounds every second at 100% interest for a year?
Is there a way to get even low than a second?
Also, look at infinite by having a student walk half way across classroom by moving ½ the distance each move. See page 150.
Introduce new formula: $B = pe^{rt}$

When I observe students: Students are talking to table mates, raising hands to guess. They might be using $B = p\left(1+\frac{r}{n}\right)^{nt}$ and trying to figure out what measurement of time they could use to compound.

Questions to Develop Mathematical Thinking as you observe: How do you calculate infinity? Is there a way to capture infinity in math? What is the lowest measurement of time you could use to calculate compounding?

Answers: You will have $2,718.28 after a year of continuous compounding or when you compound by second.

Summarize: What techniques did you use to try to solve the $1000 compounding continuously for a year?
Practice: Day 1 pg 154-155 #1-9 odd, Day 2 pg 154-155 #2-10 even
Exit Ticket: Write 3 things you learned so far in chapter 3. Write 2 things you are struggling with in chapter 3. Write 1 thing you don't understand in chapter 3.

Career Application(s):

Banker	Venture Capitalist	Economist
Builder	Investor	Insurance Agents
Real Estate Agents	Lawyer	Doctor
Teacher	Laborer	

21st Century Skills and Interdisciplinary Themes:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness ☒ Financial/Economic/Business/Entrepreneurial Literacy ☐ Civic Literacy
☐ Health/Safety Literacy ☐ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION
Creativity and Innovation
☒ Think Creatively
☒ Work Creatively with Others
☐ Implement Innovations
Critical Thinking and Problem Solving
☒ Reason Effectively
☒ Use Systems Thinking
☐ Make Judgments and Decisions
☒ Solve Problems
Communication and Collaboration
☒ Communicate Clearly
☒ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS
Information Literacy
☒ Access and Evaluate Information
☒ Use and manage Information
Media Literacy
☐ Analyze Media
☐ Create Media Products
Information, Communications and Technology (ICT Literacy)
☒ Apply Technology Effectively

LIFE & CAREER SKILLS
Flexibility and Adaptability
☐ Adapt to Change
☒ Be Flexible
Initiative and Self-Direction
☒ Manage Goals and Time
☐ Work Independently
☐ Be Self-Directed Learners
Social and Cross-Cultural
☒ Interact Effectively with Others
☒ Work Effectively in Diverse Teams

Productivity and Accountability
☒ Manage Projects
☒ Produce Results
Leadership and Responsibility
☐ Guide and Lead Others
☐ Be Responsible to Others

Name: _____ Date: _____ Period: _____

Homework Quiz 3-6 Continuous Compounding

1) Isaac is opening a CD to save for college. He is considering a 4 year CD or a 4 ½ year CD since he will start college around that time. He needs to be able to have the money to make tuition payments on time, and he does not want to have to withdraw money early from the CD and face a penalty. His grandma gave him $15,000 to deposit.
   a. How much interest would he earn at 4.2% compounded monthly for 4 years?

   b. How much interest would he earn at 4.2% compounded continuously for 4 ½ years?

   c. Isaac decides on a college after opening the 4 ½ year CD, and the college needs the first tuition payment a month before the CD matures. Isaac must withdraw money from the CD early, after 4 years and 5 months. She faces two penalties. First the interest rate for the last five months of the CD was lowered to 2% and the compounding changed to compounded per month. Additionally, there was a $250 penalty. Find the total interest on the 4 ½ year CD after 4 years and 5 months.

   d. The interest is reduced by subtracting the $250 penalty. What does the account earn for 4 years and 5 months?

   e. Find the balance on the CD after she withdraws $12,000 after 4 years and five months.

   f. The final month of the CD receives 2% interest. What is the final month's interest?

   g. What is the total interest for the 4 ½ year CD?

   h. Would Isaac have been better off with the 4 year CD?

Name: Key Date: _____ Period: _____

Homework Quiz 3-6 Continuous Compounding

1) Isaac is opening a CD to save for college. He is considering a 4 year CD or a 4 ½ year CD since he will start college around that time. He needs to be able to have the money to make tuition payments on time, and he does not want to have to withdraw money early from the CD and face a penalty. His grandma gave him $15,000 to deposit.

   a. How much interest would he earn at 4.2% compounded monthly for 4 years? **$2,738.85**

   $$B = p\left(1+\frac{r}{n}\right)^{nt}$$

   p=15,000
   r=0.042
   n=12
   t=4

   b. How much interest would he earn at 4.2% compounded continuously for 4 ½ years? **$3,120.61**

   $$B = pe^{rt}$$

   p=15,000
   r=0.042
   t=4.5

   c. Isaac decides on a college after opening the 4 ½ year CD, and the college needs the first tuition payment a month before the CD matures. Isaac must withdraw money from the CD early, after 4 years and 5 months. She faces two penalties. First the interest rate for the last five months of the CD was lowered to 2% and the compounding changed to compounded per month. Additionally, there was a $250 penalty. Find the total interest on the 4 ½ year CD after 4 years and 5 months.
   Interest Earned = **$2,642.41**

   $B = pe^{rt}$ for 4 years
   p=15,000
   r=0.042
   t=4
   B= **$17,744.05**

   $B = p\left(1+\frac{r}{n}\right)^{nt} - 250$ for the last 5 months
   p= $17,744.05
   r=0.02
   t= convert to fraction of a year $\frac{5}{12}$ then to decimal = 0.416666666
   n= 12
   B=$17,642.41

   d. Find the balance on the CD after he withdraws $12,000 after 4 years and five months.
   $17,642.41 – $12,000 = **$5,642.41**

   e. The final month of the CD receives 2% interest compounded monthly. What is the final month's interest? Final Month's Interest = **$9.40**

   $$B = p\left(1+\frac{r}{n}\right)^{nt}$$

   p=5642.41
   r=.02
   n=12
   t=convert to fraction of a year $\frac{1}{12}$ then to decimal = 0.08333333333

   f. What is the total interest for the 4 ½ year CD?
   $2,642.41 + $9.40 = **$2,651.81**

   g. Would Isaac have been better off with the 4 year CD? **Yes, he would have earned $87.04 more**
   $2,738.85 - $2,651.81 = $87.04