WAMC Lab Template

Math Concept(s): Unit 2: Sequences and Series
Source / Text: Illustrative Mathematics
Developed by: Illustrative Mathematics
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Date: Summer Conference 2023

Attach the following documents:

- Lab Instructions: (Attached)
- Student Handout(s):
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

Lab Plan

Lab Title: Tower of Hanoi

Prerequisite skills: Write equations, solve equations,

Lab objective: to discover the minimum number of moves to complete the tower of Hanoi, and to write the equation to calculate the minimum number of moves for any number of pieces.

Standards: *(Note SPECIFIC relationship to Science, Technology, and/or Engineering)*

Mathematics K–12 Learning Standards:

HS.A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

HS.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

HS.F.BF.1 Write a function that describes a relationship between two quantities.*

HS.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Standards for Mathematical Practice:

MP1 Make sense of problems and persevere in solving them.

MP3 Construct viable arguments and critique the reasoning of others.

MP4 Model with mathematics.

MP5 Use appropriate tools strategically.

MP7 Look for and make use of structure.

MP8 Look for and express regularity in repeated reasoning.

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- Speaking and listening. Comprehensions and collaboration
- B Work with peers to set rules for collegial discussion and decision making
- Propel conversation by posing and responding to questions that relate to the current information

Technology

- Computational thinker. Students develop and employ strategies for understanding problems that leverage the power of technology.

Leadership/21st Century Skills:

Collaborate with Others 3.B.1 Demonstrate ability to work effectively and respectfully with diverse teams - Students work together to find a pattern they see in a real world and collaborate on how they can represent the pattern mathematically

Use and Manage Information 4.B.1 Use information accurately and creatively for the issue or problem at hand - Students have to identify patterns to create equations

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness ☐ Financial/Economic/Business/Entrepreneurial Literacy ☐ Civic Literacy

☐ Health/Safety Literacy ☐ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

Creativity and Innovation

☐ Think Creatively

☐ Work Creatively with Others

☐ Implement Innovations

Critical Thinking and Problem Solving

☒ Reason Effectively

☒ Use Systems Thinking

☐ Make Judgments and Decisions

☒ Solve Problems

Communication and Collaboration

☐ Communicate Clearly

☒ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

Information Literacy

☐ Access and Evaluate Information

☐ Use and manage Information

Media Literacy

☐ Analyze Media

☐ Create Media Products

Information, Communications and Technology (ICT Literacy)

☒ Apply Technology Effectively

LIFE & CAREER SKILLS

Flexibility and Adaptability

☐ Adapt to Change

☐ Be Flexible

Initiative and Self-Direction

☐ Manage Goals and Time

☒ Work Independently

☐ Be Self-Directed Learners

Social and Cross-Cultural

☒ Interact Effectively with Others

☐ Work Effectively in Diverse Teams

Productivity and Accountability

☐ Manage Projects

☐ Produce Results

Leadership and Responsibility

☒ Guide and Lead Others

☐ Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- (https://www.transum.org/Maths/Investigation/Tower_Of_Hanoi/Default.asp?Level=1)
- Paper/pencil or other way to record data

Set-Up Required:

- None

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

Cooperative Learning: Students will share how they got the lowest moves with their team/partner, share data,

Expectations: Thought they may not be able to write the recursive sequence, they will see the pattern. This can be revisited after the official lesson, and can continue to come back until they can answer the "end of the universe" question.

Timeline:

- 30 minutes

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- Follow up questions:

  Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?

  Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?

  Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.

  How many moves do you think it will take to complete a puzzle with 5 discs?

  How many moves do you think it will take to complete a puzzle with 7 discs?

  A legend says that a Tower of Hanoi puzzle with 64 discs is being solved, one move per second. How long will it take to solve this puzzle?

Career Applications

- statistics, biology, programming, finance

Optional or Extension Activities

- Can you create a similar game?
- What is the largest number of disks that is solvable within a class period? Show your thinking.
-

https://www.transum.org/Maths/Investigation/Tower_Of_Hanoi/Default.asp?Level=1

Three simple rules are followed:

1. Only one disk can be moved at a time.
2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack. In other words, a disk can only be moved if it is the uppermost disk on a stack.
3. No larger disk may be placed on top of a smaller disk.

Group the students into teams of four. Have then pick or assign a roll for each student in the team.

1. Team leader and Speaker: will share their teams progress with the class
2. Records keeper: Will create table of all teams moves/and time
3. Time keeper: Keep team on track, and ensure all steps are completed
4. Team liaison will share and collect data from other teams

- The students go to the website, and complete the tower starting with level one, and moving up until they cannot complete the puzzle individually at first.

-Have them share within their table teams how they completed each puzzle, and have them record the number of moves that it takes to complete each stage.

-The team liaison will go and check in around the room with the records keeper to compile data for the whole class to bring back to their team

-Pick a team leader to share their data with the class, see what the fewest moves possible.

-optional: have students show how they solved each stage

-Write down in the table for all to see the stage and the min number of moves, ask for patterns that they see

-ask each team to try find out the minimum number of moves for a tower with 8 discs,

It is ok if they do not have the equation, this can be discussed after the official lesson on recursive sequences. They should want to explore the topic and the puzzle

-Pass out reflection questions/ share formulas and equations if the class was able to create any (share if they are correct or not, ask if they can be verified by the tables that we have all created)

<u>reflection questions</u>

If we are using 3 discs what is the smallest number of moves you can find? What about 4 discs?

We can use the solution for 3 discs to solve the puzzle for 4 discs. Discuss in your teams how this might happen.

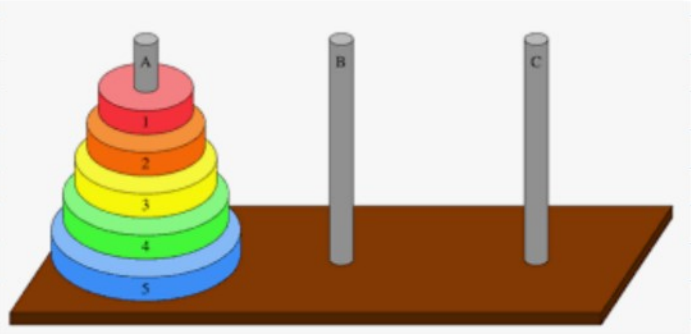
How many moves do you think you can complete a puzzle with 15 discs? What is the minimum number of moves?
What was easy about this lab?

What was challenging?

Sequences and Series Introduction: Tower of Hanoi

Tuesday, June 27, 2023 1:19 PM

In the Tower of Hanoi puzzle, a set of discs sits on a peg, while there are 2 other empty pegs.



A *move* in the Tower of Hanoi puzzle involves taking a disc and moving it to another peg. There are two rules:

- Only move 1 disc at a time.
- Never put a larger disc on top of a smaller one.

You complete the puzzle by building the complete tower on a different peg than the starting peg.

Tower of Hanoi

Number of discs	Number of moves (first try)	Minimum number of moves possible
1		
2		
3		
4		
5		
6		

1. Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?

2. Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?

3. Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.

4. How many moves do you think it will take to complete a puzzle with 5 discs? Explain or show your reasoning.

There is a legend that there is a set of 64 gold disks on 3 diamond needles called the *Tower of Brahma* which monks have been trying to solve since the beginning of time.

Supposedly the Universe will end when they succeed in moving the 64 discs from the start needle to the end needle without breaking any of the rules.

Can you work out when the universe will end? (assuming that they make no mistakes and move one piece per second)

Answer:

Completed table

Number of discs	Number of moves (first try)	Minimum number of moves possible
1		1
2		3
3		7
4		15
5		31
6		63
7		127
8		255
9		511

Or $2^n - 1$

Application of Geometric Sequence

Wednesday, August 31, 2022 1:46 PM

Learning Targets

- *Describe and graph geometric sequences.*
- *Write a recursive formula for a geometric sequence.*
- *Write and explicit formula for a geometric sequence.*

How Much Revenue?

The new start-up is thriving! More and more customers are trying out the products, leading to increased sales. The company made $20,000 in revenue in their first month, and this amount is growing by 3% each month.

1. Complete the table.

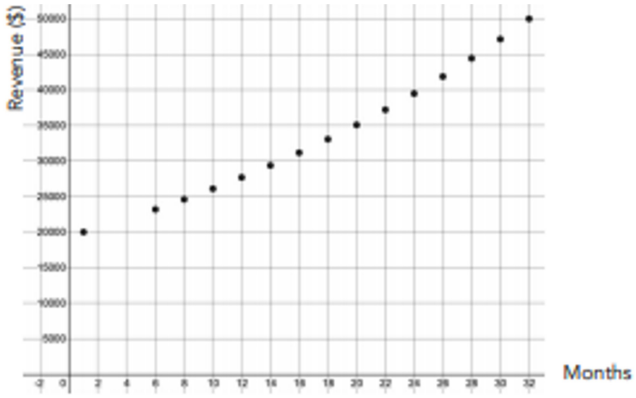
	Revenue ($)
Month 1	20,000
Month 2	
Month 3	
Month 4	

2. Is the revenue increasing by the same dollar amount each month? How do you know?

3. Write a recursive rule to figure out the new month's revenue if you know the previous month's revenue.

4. Predict the company's revenue in month 11. Show your method.

5. A graph of the monthly revenue is shown. Add your ordered pairs from the table. What do you notice about the shape of the graph?



6. Suppose that after many months, the company's revenue starts *decreasing* by 3%. How would this affect your calculations?

MATH MEDIC

Lesson 1.4 – Applications of Geometric Sequences

QuickNotes

Check Your Understanding

1. Quinten has a secret. On the first day of school, he tells his closest friend. On the second day, they each tell the secret to someone else. Each person that hears the secret shares the secret with a new person each day.
    a. How many people will know the secret on the 4^{th} day of school?

    b. If on the 10^{th} day of school, 1024 people know the secret, how many will know the secret by the 11^{th} day?

    c. Write an explicit rule for the number of students who know the secret on day *n*.

2. Which of the following are an example of geometric decay? Choose all that apply.

2. Which of the following are an example of geometric decay? Choose all that apply.

   _____ A bank account earns 1.25% interest annually

   _____A shirt is on sale for 15% off.

   _____A swimmer reduces his 50-m backstroke by 2% each week of the summer

   _____A teacher hands out 2 pencils every day

3. Could this table of values represent a geometric sequence? Why or why not?

n	1	2	3	4	5
a_n	18	24	32	42	54

4. A new iPhone is purchased for $799. The value of the iPhone decreases by 33% each year. How much is the iPhone worth 5 years after it was purchased?

There is a legend of Tower of Hanoi puzzle with 64 discs that is being solved with one move per second. How long do you think it will take to solve this puzzle? Show your thinking.

Name_____

Date_____

Period_____

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