WAMC Lab Template

Math Concept(s): Exponential Functions, Growth and Decay

Source / Text: none

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Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

- Growth: Students estimate and then simulate the number of beans when doubling 8 times, then multiplying by $\frac{3}{2}$ each time (rounding as needed), and multiplying by $7/4$ each time. On a piece of paper, they circle each pile of beans and record the value inside. Data is then recorded on a table, and plotted using Desmos.
- Have a gallery walk or group sharing to compare results, then address vocabulary needs.
- Decay 1: Students begin with an initial number of beans (different for each group?), and remove a fraction $\left(\frac{2}{3}, \frac{3}{4}, \frac{2}{5}\right)$ of the population each time, leaving piles behind. On a piece of paper, they circle each pile of beans and record the value inside. Data is then recorded on a table, and plotted using Desmos.
- Equations: Use the Desmos regression to write the equations of each of their patterns. Explain how each value in the equation affects the outcomes, for the circles quantities, the data tables, and the graphs. Discuss why the model does not precisely match – what decisions in the process led to this result?
- Have another gallery walk, and a class discussion to share the Part III: Hill of Beans portion. Briefly explain that R^2 is a measure of how closely the data fits, with 1 being a perfect match between the theoretical value and the measured value. Focus on concepts such as “repeated multiplication” and exponential growth or decay, and reinforce vocabulary usage using a shared written space. Allow students to disagree, so long as they can explain. Avoid going into discussion and extensions at this time, and make sure students have anchored the growth $b > 1$ and decay $0 < b < 1$ concept.
- Decay 2: Students begin with an initial number of beans (different for each group?), and then attempt to get as close to exactly 1 as they can after 2, 4, and 8 reductions of the same proportion. They need to determine the value of the proportion with the closest approximation, then graph their data points. Finally, they use the exponential regression $y_1 \sim a(b^{x_1})$ to compare their solutions. Be sure to circulate during this process, as students may get frustrated when the values don’t work perfectly.
- Application: Students research exponential decay online, and write a short narrative replacing the beans from their data with a real world analog.

Lab Plan
Lab Title: Functions with BEANS

Prerequisite skills:
Counting
Multiplying by fractions (calculator okay)
Rounding
Entering tables and point on Desmos

Lab objective:
Demonstrate effects of exponential growth and decay in systems
Develop concepts of initial value, growth/decay rates
Establish the concept of logarithms as a method of solving for unknowns in exponents
Apply exponential and logarithmic patterns to multiple real world applications

Standards: *(Note SPECIFIC relationship to Science, Technology, and/or Engineering)*
Mathematics K–12 Learning Standards:

- A.SSE.A.1.A - Interpret parts of an expression, such as terms, factors, and coefficients.
- A.SSE.B.3 - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- F.LE.A.1.C - Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
- F.LE.A.2 - Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F.LE.A.3 - Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- F.LE.B.5 - Interpret the parameters in a linear or exponential function in terms of a context.
- F.BF.B.5 - Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Standards for Mathematical Practice:

- 1 – Make sense of problems and persevere in solving them.
- 2 – Reason abstractly and quantitatively
- 3 – Construct viable arguments and critique the reasoning of others.
- 4 – Model with mathematics.
- 5 – Use appropriate tools strategically.
- 6 – Attend to precision.
- 7 – Look for and make use of structure.

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- ELA-LITERACY.RST.11-12.3 - Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
- ELA-LITERACY.RST.11-12.4 - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
- ELA-LITERACY.RST.11-12.8 - Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
- ELA-LITERACY.SL.11-12.1 - Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 11-12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
- ELA-LITERACY.SL.11-12.4 - Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.
- ELA-LITERACY.W.11-12.2.D - Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic.
- ELA-LITERACY.W.11-12.3 - Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

K-12 Science Standards

- ESS1-6. Apply scientific reasoning and evidence from ancient Earth materials, meteorites, and other planetary surfaces to construct an account of Earth's formation and early history.
- PS1.C: Nuclear Processes Nuclear processes, including fusion, fission, and radioactive decays of unstable nuclei, involve release or absorption of energy. The total number of neutrons plus protons does not change in any nuclear process.
- Scale, Proportion, and Quantity - Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
- LS1.B: Growth and Development of Organisms - In multicellular organisms individual cells grow and then divide via a process called mitosis, thereby allowing the organism to grow. The organism begins as a single cell (fertilized egg) that divides successively to produce many cells, with each parent cell passing identical genetic material (two variants of each chromosome pair) to both daughter cells. Cellular division and differentiation produce and maintain a complex organism, composed of systems of tissues and organs that work together to meet the needs of the whole organism.

- LS2.A: Interdependent Relationships in Ecosystems - Ecosystems have carrying capacities, which are limits to the numbers of organisms and populations they can support. These limits result from such factors as the availability of living and nonliving resources and from such challenges such as predation, competition, and disease. Organisms would have the capacity to produce populations of great size were it not for the fact that environments and resources are finite. This fundamental tension affects the abundance (number of individuals) of species in any given ecosystem.
- ESS3.C: Human Impacts on Earth Systems - The sustainability of human societies and the biodiversity that supports them requires responsible management of natural resources.

Technology (EdTech)

- Knowledge Constructor 3.a. Students plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.
- Knowledge Constructor 3.d. Students build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
- Innovative Designer - 4.d. Students exhibit a tolerance for ambiguity, perseverance and the capacity to work with open-ended problems.
- Computational Thinker - 5.b. Students collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
- Creative Communicator - 6.c. Students communicate complex ideas clearly and effectively by creating or using a variety of digital objects such as visualizations, models or simulations.

Engineering (ISTE)

- 1.3.d Students build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.
- 1.5.b Students collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.
- 1.7.c Students contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness ☐ Financial/Economic/Business/Entrepreneurial Literacy ☐ Civic Literacy
☐ Health/Safety Literacy ☐ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION
Creativity and Innovation
☒ Think Creatively
☒ Work Creatively with Others
☐ Implement Innovations
Critical Thinking and Problem Solving
☒ Reason Effectively
☒ Use Systems Thinking
☒ Make Judgments and Decisions
☒ Solve Problems
Communication and Collaboration
☒ Communicate Clearly
☒ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS
Information Literacy
☐ Access and Evaluate Information
☐ Use and manage Information
Media Literacy
☐ Analyze Media
☐ Create Media Products
Information, Communications and Technology (ICT Literacy)
☒ Apply Technology Effectively

LIFE & CAREER SKILLS
Flexibility and Adaptability
☐ Adapt to Change
☒ Be Flexible
Initiative and Self-Direction
☒ Manage Goals and Time
☒ Work Independently
☒ Be Self-Directed Learners
Social and Cross-Cultural
☒ Interact Effectively with Others
☒ Work Effectively in Diverse Teams

Productivity and Accountability
☒ Manage Projects
☒ Produce Results
Leadership and Responsibility
☐ Guide and Lead Others
☒ Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Large sack of beans
- Sandwich bags
- Access to Desmos
- Paper (standard) and pencil
- Topics/possible sites to assign for deeper research

Set-Up Required:

- Count out approximately 250 beans for each group
- Collect paper to hand out
- Determine appropriate groups of 3-4
- Print student activity guide

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

- 3-4 students assigned to groups
- Creative thinking and problem solving skills

Cooperative Learning:

- 3-4 students assigned to groups without choice, with diverse backgrounds and skill sets
- Students will need to communicate clearly, using appropriate vocabulary
- Discussion and gallery walks will expose students to more situations than they are investigating themselves.

Expectations:

- Students will persist in solving ambiguous situations such as rounding continuous functions into discrete values, or "how close is close enough"
- Students will collect data and control their pace
- At the end of learning, students will be able to describe exponential growth and decay using the concept of repeated growth and decay, and be familiar with possible uses for the pattern.

Timeline:

- 2 days total

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- Applying repeated patterns to understand consequences is helpful to understand many different situation. In this case, students are simulating data collection using mathematical thinking that can be used in scientific and business situations. They are connecting a process of taking (or adding) a proportional amount over and over, and should be able to determine whether or not future applications fit this pattern or not.
- Trying to fit constraints such as "reach 1 within 8 turns" matches many construction or agricultural expectations.

- In general, people do not predict exponential change well, and mistake it for linear change. Students may increase their ability adjust for this internal bias through repeated experience.

Career Applications

- Data collection and analysis involving exponential growth or decay is required for STEM careers and many business applications. The first portion applies to financial decisions and populations, while the second portion explores decaying values such as depreciation, half-life, and many properties of medications.

Optional or Extension Activities

- Students could investigate careers that are relevant while researching applications, and check the training/education requirements and potential pay.
- Students could explore other proportions, different initial values, different number of repetitions, or a different target number than 1.
- Rather than guess and check, students could explore using Desmos parameters to determine proportions using sliders.
- The problems can be solved using algebraic methods. Some students might be able to reason it out the process using already known principles.
- A different question can be asked: how many repetitions would it take to reach a given number of beans, provided you know the proportion b. This sets up the concept of logarithms, which will be addressed later.

Functions with BEANS

Materials:

- Groups of 3-4
- Several sheets of paper
- Pencil/pen
- Chromebook for Desmos
- 1 bag of beans per group

Procedures

Before we start

1. Collect your materials and prepare your workspace.
2. Count your beans. # of beans: _____

MORE BEANS!

Part I: MORE BEANS! (No calculators)

1. You will be doubling the number of beans 8 times. **Estimate** the number of beans you will need.
   # of beans: _____ Will you have enough to double 8 times? _____

2. Put one bean on a sheet of paper. Draw a circle around it. Write the number 1 in the circle.
3. Repeat the following 8 times.
   a. For each bean in the current circle, add 2 beans to a new pile.
   b. Circle the pile of beans. Label the circle with the number of beans.

4. Write the total after 8 doublings: # of beans: _____

5. Was your original estimate accurate? Can you explain any differences?

Functions with BEANS

Part II: More Beans, But Less.

You will repeat the steps in Part I.

1. Instead of the $1 \rightarrow 2$ ratio, use $2 \rightarrow 3$.
   # of beans (estimate before): _____ Will you have enough to double 8 times? _____
   # of beans: _____

2. Instead of the $1 \rightarrow 2$ ratio, use $4 \rightarrow 7$.
   # of beans (estimate before): _____ Will you have enough to double 8 times? _____
   # of beans: _____

Part III: Beans On A Table

1. Create a table on Desmos for each of your data sets. For x_1, use the sequence $\{0,1,2,3,4,5,6,7,8\}$. For y_1, use your # of beans.

2. Type the following expression to fit a line through your data: $y_1 \sim a\ b^{x_1}$

3. Record the parameters:

Ratio	a	b	R^2
1: 2			
2: 3			
4: 7			

Functions with BEANS

LESS BEANS!

Part I: Think of a clever name

1. Begin with a pile of **all** of your beans. Draw a circle around the pile.
2. Repeat 8 times for 2/3:
   a. Move 2/3^{rds} of the pile to a new pile.
   b. Circle the pile. Label the circle with the number of beans.
3. Repeat 8 times for 3/4: Predict: how will the final number compare to 2/3^{rds} ?
   a. Move 3/4^{ths} of the pile to a new pile.
   b. Circle the pile. Label the circle with the number of beans.
4. Repeat 8 times for 2/5: Predict: how will the final number compare the others?
   a. Move 2/5^{ths} of the pile to a new pile.
   b. Circle the pile. Label the circle with the number of beans.

Part II: Beans On A Table Again

1. Create a table on Desmos for each of your data sets. For x_1, use the sequence $\{0,1,2,3,4,5,6,7,8\}$. For y_1, use your # of beans.
2. Type the following expression to fit a line through your data: $y_2 \sim a\ b^{x_2}$
3. Record the parameters:

Fraction	**Final # of Beans**	a	b	R^2
$\frac{2}{3}$				
$\frac{3}{4}$				
$\frac{2}{5}$				

Functions with BEANS

Part III: A Hill Of Beans – Discussion Prep

1. What effect does changing the fraction have on the final amount?

2. Write a rule (or rules) describing the effect of fractions.

3. Look at your tables of parameters.
   a. What value does a probably represent?

   b. What value does b probably represent?

4. Do the amounts in a pile follow a straight line? If so, what would its equation be?

5. Will the number of beans ever be 0?

6. Will you ever need a negative number of beans?

7. Why don't the lines go perfectly through your data points?

8. With your group, consider the tables, the circles, and the graphs. Write 2-3 notice or wonders about the results.

1	
2	
3	
4	

Functions with BEANS

MORE LESS BEANS!

Part I: Bean Decay

1. Begin with a pile of all of your beans. # of beans: _____
2. Find the proportion to remove each time to have as close to 1 bean as possible **after 2 repetitions**.
3. Find the proportion to remove each time to have as close to 1 bean as possible **after 4 repetitions**.
4. Find the proportion to remove each time to have as close to 1 bean as possible **after 8 repetitions**.
5. For each, graph your data on Desmos and record in the table.

Repetitions	**Final # of Beans**	a	b	R^2
2				
4				
8				

6. Write the equations of the patterns you used.
7. Do the amounts in a pile follow a straight line? How can you tell?
8. What does x represent on your graph?
9. Will the number of beans ever be 0?
10. Will you ever need a negative number of beans?
11. Why don't the lines go perfectly through your data points?

Functions with BEANS

Part II: There Are No Beans

Your teacher will provide you with some possible topics to research. After reading, use your data as if it were collected in the real world; replace BEANS with something in your topic, and replace x with a unit of measurement.

Functions with BEANS

Rubric

Student Name: _____

Group: _____

Category	Score				Comments
Growth – MORE BEANS Data collected	4	3	2	1	
Growth – MORE BEANS Values estimated	4	3	2	1	
Growth – MORE BEANS Worked with team	4	3	2	1	
Decay – LESS BEANS Data collected	4	3	2	1	
Decay – LESS BEANS Discussion Prep	4	3	2	1	
Decay – LESS BEANS Worked with team	4	3	2	1	
Decay – MORE LESS BEANS Solved proportions	4	3	2	1	
Decay – MORE LESS BEANS Questions Answered	4	3	2	1	
Decay – MORE LESS BEANS Research done	4	3	2	1	
Decay – MORE LESS BEANS Research done	4	3	2	1	

Strengths	Challenges	Possible Approaches in the Future

WAMC Lesson Plan

Name(s): Nick Genereux
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Lesson Title: Exponential Beans Lab
Date: 6/22/2022
Text: STEM Correlation: Lesson Length: 4 days

Big Idea (Cluster): Functions – Linear, Quadratic, and Exponential Models

Mathematics K–12 Learning Standards:

- F.LE.A.1.C - Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
- F.LE.A.2 - Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F.LE.A.3 - Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- F.LE.B.5 - Interpret the parameters in a linear or exponential function in terms of a context.

Mathematical Practice(s):

- 1 – Make sense of problems and persevere in solving them.
- 2 – Reason abstractly and quantitatively
- 3 – Construct viable arguments and critique the reasoning of others.
- 4 – Model with mathematics.
- 5 – Use appropriate tools strategically.
- 6 – Attend to precision.
- 7 – Look for and make use of structure.

Content Objectives:	Language Objectives (ELL):
Students will be able to explain why certain patterns increase or decrease exponentially, using rational bases	Students will develop fluency with terms used to describe exponential functions, including growth, base, exponent, rates, and parameters.
Vocabulary: Parameter Rate Exponential growth Exponential decay	Connections to Prior Learning Students have multiplied by fractions, used t-tables to collect data, and should have some experience with linear regression.
Questions to Develop Mathematical Thinking: • Will these pattern increase or decrease? • What values did you use to get that result? • How does this result match your prediction? Does that seem reasonable? • How is this different than a linear pattern?	Common Misconceptions: • Fractions/decimals can't have exponents, or mistakes when using them. • Removing a proportion versus leaving a proportion • Exponents always make values bigger.

Assessment (Formative and Summative):

- Students will complete an activity packet, which allows for informal assessment and remediation during the procedures. Teachers should assess whether responses are reasonable, and encourage more explanation whenever possible.
- A rubric for teacher notes and observations is provided for the lab.
- Practice sets for specific skills, including teacher-guided and independent work
- A formative assessment using representative demonstration of skills practiced by students, as well as conceptual questions for students to respond.

Materials:

- Between 200 and 300 beans in bags for each group
- Student handouts
- Student assessment rubric
- Plain paper – 6-8 per group
- Practice sets and skill-focused lessons

Instruction Plan:

Introduction: A vocabulary poster for the unit, including major features and pronunciation, following the district vocabulary practices.

Explore: Conduct the Functions with BEANS lab
Expand on concepts to apply algebra skills using practice sets, connecting back the lab each class period. Complete and interact on vocabulary poster daily.

When I observe students:
Students should be engaged actively at all times, usually discussing or moving

Questions to Develop Mathematical Thinking as you observe:

- Will these pattern increase or decrease?
- What values did you use to get that result?
- How does this result match your prediction? Does that seem reasonable?

Answers:

- Consider the base: if $b > 1$ then the function grows, if $0 < b < 1$ then the function decays.
- Identify specific expressions in student work, either written or in the calculator. Encourage students to show their reasoning.
- "My prediction was close to the correct answer…" "My prediction was not close, because I was imagining a linear model, used the wrong fraction, or made mistake when calculating."

Summarize:

-

Career Application(s):

- Data collection and analysis involving exponential growth or decay is required for STEM careers and many business applications. The first portion applies to financial decisions and populations, while the second portion explores decaying values such as depreciation, half-life, and many properties of medications.

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☒ Global Awareness ☒ Financial/Economic/Business/Entrepreneurial Literacy ☐ Civic Literacy
☒ Health/Safety Literacy ☒ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

Creativity and Innovation
- ☐ Think Creatively
- ☐ Work Creatively with Others
- ☐ Implement Innovations

Critical Thinking and Problem Solving
- ☒ Reason Effectively
- ☒ Use Systems Thinking
- ☒ Make Judgments and Decisions
- ☒ Solve Problems

Communication and Collaboration
- ☒ Communicate Clearly
- ☒ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

Information Literacy
- ☐ Access and Evaluate Information
- ☒ Use and manage Information

Media Literacy
- ☐ Analyze Media
- ☐ Create Media Products

Information, Communications and Technology (ICT Literacy)
- ☐ Apply Technology Effectively

LIFE & CAREER SKILLS

Flexibility and Adaptability
- ☐ Adapt to Change
- ☒ Be Flexible

Initiative and Self-Direction
- ☒ Manage Goals and Time
- ☐ Work Independently
- ☒ Be Self-Directed Learners

Social and Cross-Cultural
- ☒ Interact Effectively with Others
- ☒ Work Effectively in Diverse Teams

Productivity and Accountability
- ☒ Manage Projects
- ☒ Produce Results

Leadership and Responsibility
- ☐ Guide and Lead Others
- ☒ Be Responsible to Others