WAMC Lab Template
Math Concept(s): Nonlinear Equations
Source / Text:
Developed by: Michael Barnhart E-Mail: Michael.barnhart.15@gmail.com
Date: Summer Conference 2023

Attach the following documents:

- Lab Instructions: See attached
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):
This activity has the students collect data to help them create a graph showing the relationship between the drop height of a ball and how long it bounces. This lab takes place at the end of instruction on "Nonlinear Relationships." This hands-on activity provides students an opportunity to apply their knowledge about nonlinear relationships through the process of collecting data, graphing their data, and identifying if the relationship between two variables is linear or nonlinear.

Lab Plan
Lab Title: Height and time of bouncing ball

Prerequisite skills: Students should have basic knowledge of nonlinear equations and creating and graphing data involving nonlinear equations.

Lab objective: The objective of this lab is to help students apply their knowledge of nonlinear equations to explain the affect drop height has on how long a racquetball bounces.

Standards: ***(Note SPECIFIC relationship to Science, Technology, and/or Engineering)***
Mathematics K–12 Learning Standards:

- HSF-IF.A: Understand the concept of a function and use of function notation
- HSF-IF.B: Interpret functions that arise in applications in terms of the context
- HSN-IF.C: Analyze functions using different representations
- HSF-ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables

Standards for Mathematical Practice:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Model with mathematics
- Use appropriate tools strategically

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- Speaking and listening. Comprehension and Collaboration
- Work with peers and set rules for collegial discussions and decision making
- Propel conversations by posing and responding to questions that relate to the current information

K-12 Science Standards

- HS-PS2-1. Motion and Stability

Technology

- 5.b Students collect data or identify relevant data sets, use digital tools to analyze them and represent data in various ways to facilitate problem solving and decision making.

Engineering

- HS-ETS 1-3. Engineering Design

Leadership/21st Century Skills:

<u>21st Century Interdisciplinary themes</u> (Check those that apply to the above activity.)

☐ Global Awareness
☐ Financial/Economic/Business/Entrepreneurial Literacy
☐ Civic Literacy
☐ Health/Safety Literacy
☐ Environmental Literacy

<u>21st Century Skills</u> (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

<u>Creativity and Innovation</u>
☐ Think Creatively
☒ Work Creatively with Others
☐ Implement Innovations

<u>Critical Thinking and Problem Solving</u>
☐ Reason Effectively
☐ Use Systems Thinking
☒ Make Judgments and Decisions
☒ Solve Problems

<u>Communication and Collaboration</u>
☐ Communicate Clearly
☐ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

<u>Information Literacy</u>
☐ Access and Evaluate Information
☐ Use and manage Information

<u>Media Literacy</u>
☐ Analyze Media
☐ Create Media Products

<u>Information, Communications and Technology (ICT Literacy)</u>
☐ Apply Technology Effectively

LIFE & CAREER SKILLS

<u>Flexibility and Adaptability</u>
☐ Adapt to Change
☐ Be Flexible

<u>Initiative and Self-Direction</u>
☒ Manage Goals and Time
☐ Work Independently
☐ Be Self-Directed Learners

<u>Social and Cross-Cultural</u>
☒ Interact Effectively with Others
☐ Work Effectively in Diverse Teams

<u>Productivity and Accountability</u>
☐ Manage Projects
☒ Produce Results

<u>Leadership and Responsibility</u>
☐ Guide and Lead Others
☒ Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Rubber ball such as a racquetball
- Tape measure
- Timer
- Calculator
- Paper and pencil
- Chairs (to help with dropping the ball from the maximum height)

Set-Up Required:

- Classroom or another room that has a floor surface where a ball is able to bounce for a period of time.

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

Cooperative Learning:

- Students will be working in groups of 3-5 people. One person to hold the tape measure, one person to drop the ball, one person to time how long the ball bounces, one person to record the times on paper, and one person to calculate. If groups of 5 people cannot be formed, then a group member may need to double as the timer and recorder.

Expectations:

- It is expected that students will gain an understanding on how nonlinear equations can be created out of data that gathered from hands-on activities. From there, students should be able to brainstorm ideas on how to impact the time the ball bounces outside of changing the drop height.

Timeline:

- This should be a lab that can be completed in 1-2 days. The ball dropping and data collection should take up to half of the first day maximum. This lab can be done in one day if students are highly skilled in plotting data points on a graph. If graphing data is a challenge for students, an extra day may be needed to complete the graphing of data and answer lab questions.

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- Students will know they must apply more force to an object in order to have that object return to its initial height. This is very relevant in sports such as basketball.

Career Applications

- Physics, athletics

Optional or Extension Activities

- This lab is very diverse. Students can experiment with a different type of ball such as a tennis ball, ping pong ball, golf ball, basketball, etc. Students can also experiment with a different surface such as marble, carpet, concrete, wood, etc. A "force" (student pushes/throws ball into ground at the start) can be applied to experiment how that impacts the bouncing time.

STUDENT INSTRUCTIONS

Statement of Problem

When a ball is dropped to the floor from a given height, the time it takes for the ball to stop bouncing depends on the height from which it is dropped. In this activity, you measure the length of time a ball bounces when you drop it from different heights.

Grouping Instructions and Roles

You will be divided into groups of 4-5 people.

One person to hold the tape measure
One person to drop the ball
One person to time how long the ball bounces
One person to record the times on paper
One person to perform calculations on the calculator.
Note: If groups of 5 people cannot be formed, then a group member may need to double as the timer and recorder.

Procedures

1. Locate an area where there is a hard, smooth floor. A tile, wood, or concrete surface should work well.
2. Drop the ball onto the floor from a height of 1 foot. Measure the length of time the ball bounces. Write the drop height and time of bounce (nearest 1/100 second) on a sheet of data paper.
3. Repeat Step 3 four more times.
4. Repeat Steps 2 and 3, using drop heights of 2 feet, 3 feet, 4 feet, 5 feet, and 6 feet.

Outcome Instructions

1. After all the data is collected and recorded, the group gathers to calculate the average time the ball bounces for each drop height. Record the drop height and the average time (rounded to nearest 1/100 second) on a sheet of data paper.
2. Plot a graph of drop height on the horizontal axis and time on the vertical axis. Make sure you select appropriate data intervals for your collected data (you will probably be okay with intervals of “1” for both of your axis).

Assessment Criteria (what will you be assessed on?)

Data collection
Accurate calculations of the average time the ball bounces
Accurate and adequate graph of the data
Correct identification of a linear or nonlinear relationship between height and bouncing time

Student Lab Data Sheet

Name: ____________________ Date: _______________ Class: _____

Lab Title: ____

Write the objective (in your own words) of the lab you are performing.

Data Collection:

Bounce Times (seconds)

Height	Drop 1	Drop 2	Drop 3	Drop 4
1 Foot				
2 Feet				
3 Feet				
4 Feet				
5 Feet				
6 Feet				

Calculations

Height	Avg. Bounce Time (seconds)
1 Foot	
2 Feet	
3 Feet	
4 Feet	
5 Feet	
6 Feet	

Graphing Data

Use your graph paper to plot the data on average bounce time. Height should be you "X-Axis" and Bounce Time should be your "Y-Axis."

Circle the correct answer: The relationship between drop height and bouncing time is…

Linear Nonlinear

Lab Grading Rubric: Height and Time of Bouncing Ball

Name: ______________________ Date: ________________ Class: _____

Topic	Points Possible	Points Earned
Data Collection	15	
Calculations of Avg. Bounce Time	10	
Graph of Data	20	
Correct linear/nonlinear identification	5	

WAMC Lesson Plan

Name(s): Michael Barnhart
Email Address: michael.barnhart.15@gmail.com
Lesson Title: Nonlinear Equations
Date: Summer 2023
Text: STEM Correlation: Physics, Math Lesson Length: 1-2 Class Periods

Big Idea (Cluster): Nonlinear equations are used in several occupations. Understanding what nonlinear equations are, and how they differ from linear equations, is important for most occupations that utilize science, technology, engineering, math.

Mathematics K–12 Learning Standards:
- HSF-IF.A: Understand the concept of a function and use of function notation
- HSF-IF.B: Interpret functions that arise in applications in terms of the context
- HSN-IF.C: Analyze functions using different representations
- HSF-ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables

Mathematical Practice(s):
- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Model with mathematics
- Use appropriate tools strategically

Content Objectives:	Language Objectives (ELL):
• Students will be able to create data tables for nonlinear equations and graph nonlinear equations.	• Students will be able to describe data and/or approach to solve a problem through visual data displays
Vocabulary: • Nonlinear • Reciprocal • Square	Connections to Prior Learning • Linear Equations • Formulas • Graphing
Questions to Develop Mathematical Thinking: • Why is it important to distinguish nonlinear relationships from linear relationships? • What are some real life situations where nonlinear relationships are relevant?	Common Misconceptions: • Identifying the correct type of function from a graph and/or situation • By creating the table of results, students will be more able to choose a suitable scale for their axis

Assessment (Formative and Summative):
- Exit Ticket: Students complete ET on their own; teacher walks around to check for understanding; review ET as a whole class (formative)
- Unit Test (summative)

Materials:
- CORD Mathematics: Pg. 2-12
- Paper for writing notes about nonlinear equations
- Pencil
- Graph Paper
- Calculator
- Worksheet
- Exit Ticket

Instruction Plan:

Introduction:
- Draw a graph and table representing a linear relationship on the whiteboard
- Draw a graph and table representing a nonlinear relationship on the whiteboard

Explore:
- Hand out “Nonlinear Equations” worksheet
- Go through at least one example (preferably a problem that is labeled “Easy” regarding its level.

When I observe students:
- Assign worksheet for students to complete in class so I’m available to assist them if they are struggling with certain problems.
- When observing students, attempt to identify what problems on the worksheet are presenting students with significant challenges that may need to be addressed with the whole class.

Questions to Develop Mathematical Thinking as you observe:
- Why is it important to distinguish nonlinear relationships from linear relationships?
- What are some real life situations where nonlinear relationships are relevant?

Answers:
- Distinguishing between linear and nonlinear relationships allows us to identify and apply the correct mathematical formulas to a relevant situation, which will lead to accurate data collection and explanation of what is happening.
- Nonlinear relationships are relevant in industries like finance where the value of money increases at a nonlinear rate when placed in certain accounts (e.g. savings, index funds). Nonlinear relationships are also important in the medical industry where nurses need to know how to correctly change the drip rates in IVs so a patient is receiving enough fluids from the IV.

Summarize:
- Students will be able to create data tables for nonlinear equations and graph nonlinear equations.

Career Application(s):
- Physics
- Athletics
- Business
- Medical

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness
☒ Financial/Economic/Business/Entrepreneurial Literacy
☐ Civic Literacy
☒ Health/Safety Literacy
☐ Environmental Literacy

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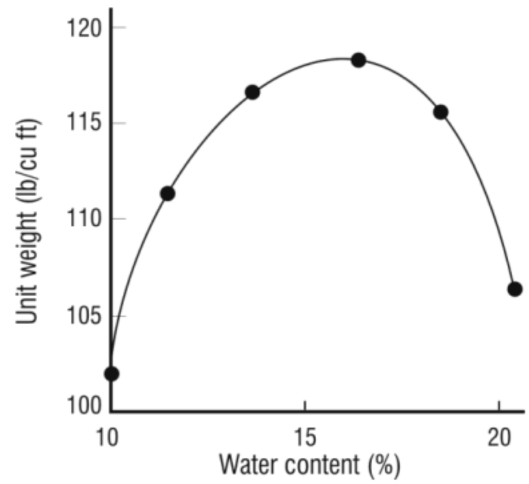
Nonlinear Equations Worksheet

Name: ______________________ Date: ________________ Class: _____

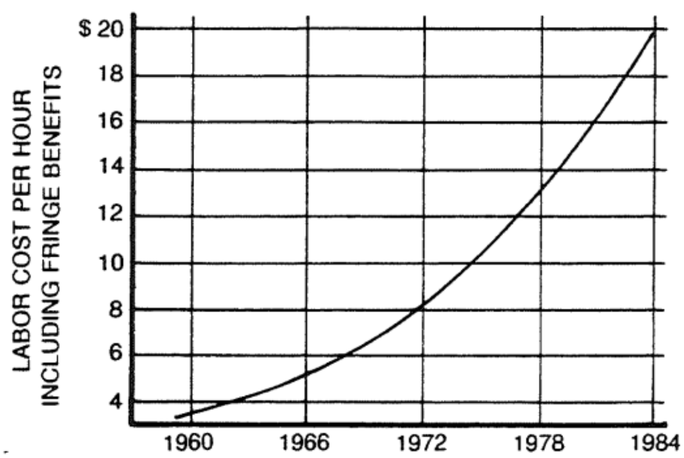
1. The water content of soil affects its weight. The results of certain soil tests produce the results shown below as a graph (called a compaction curve).



   a. Examine the curve that is drawn through the test data. Does the trend it illustrates appear linear or nonlinear?

   b. The most important feature of the curve is the maximum, that is, the high point of the peak. The maximum weight shown on the graph is called the maximum dry density, or the M.D.D. The percent water content at which this maximum occurs is called the optimum moisture, or O.M. of the soil. Approximately what are the M.D.D and the O.M. of this soil sample?

2. The graph below illustrates the changing labor costs in the American automotive industry over a 25-year period.



   a. Is this a graph of a linear trend or a nonlinear trend?

   b. The shape of the curve most resembles that of which of the following relationships:

1) $y = x^2$

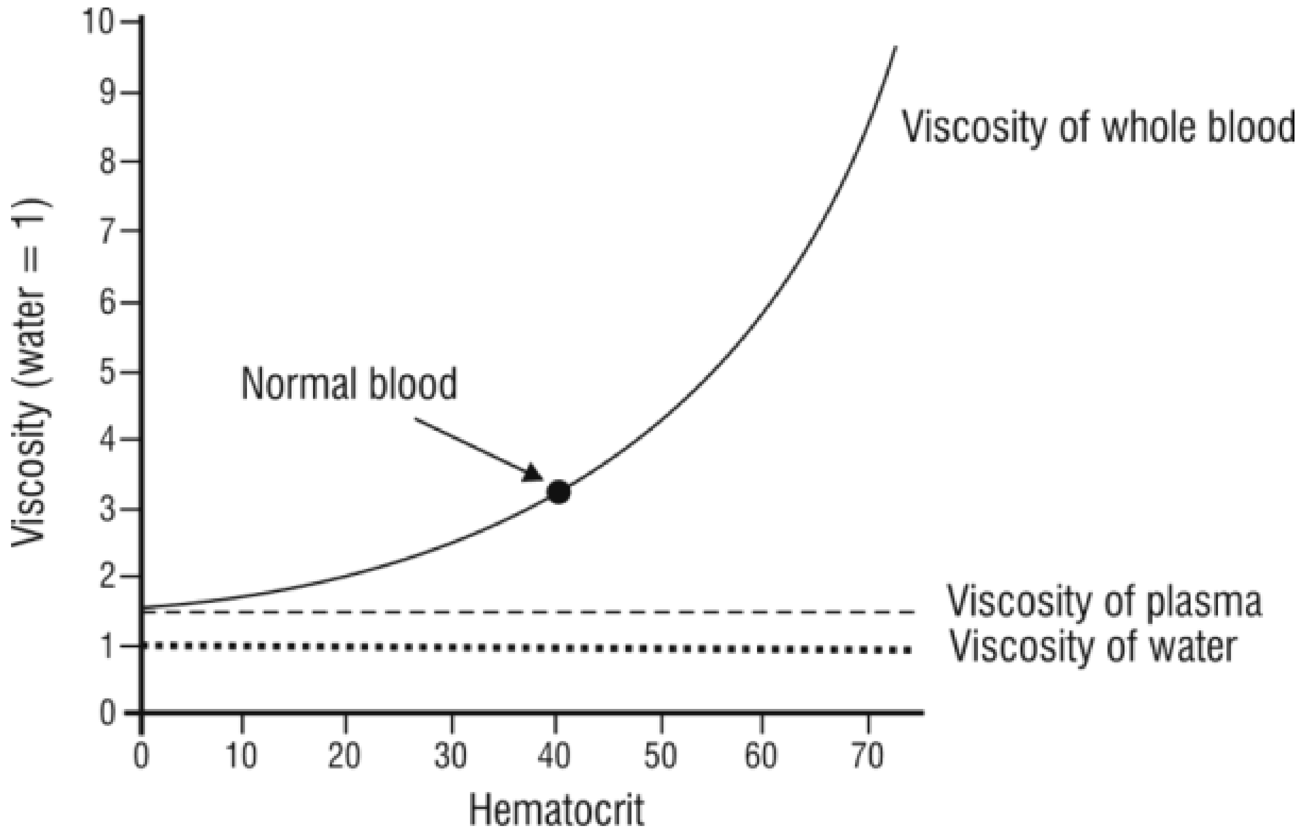
2) $y = x$

3) $y = 1/x$

Nonlinear Equations Worksheet

c. Compute the slope of the small section of the graph for the period of the 1960s, and then also for the 1970s. (Round your answer to the nearest $0.01 per hour) Did you get about the same values? Explain.

3. When the count of red blood cells increases, there is more friction between the layers of cells. This friction determines the thickness, or viscosity, of the blood. A common measure of the blood-cell count is the hematocrit—the percentage of the blood by volume that is cells. A graph of the effect of various hematocrit values is shown below. (Note that the viscosity of water is shown as equal to 1 on the graph, for comparison with whole blood and with plasma.)



a. Which of the curves in the graph appears to be nonlinear?

b. Which of the following relationships does the nonlinear graph appear to resemble:

1) $y = x^2$

2) $y = \sqrt{x}$

3) $y = {}^{1}/_{x}$

Nonlinear Equations Worksheet

c. Consider the nonlinear curve for low values of hematocrit and for higher values. In which case does a small change in the hematocrit have more effect on the viscosity?

4. The vapor pressure of water is very important when we consider breathing. A table of the vapor pressures of water (in units of millimeters of mercury) for various temperatures is shown below.

Temp (°C)	Pressure (mm Hg)
0	4.6
10	9.1
20	17.4
30	31.5
40	54.9
50	92.0
60	148.9
70	233.3
80	354.9
90	525.5
100	760.0

a. On your graph paper, draw a graph of the vapor pressure of water for the temperatures listed in the table above. List your axis appropriately (make a note that this is your graph for Problem 4).

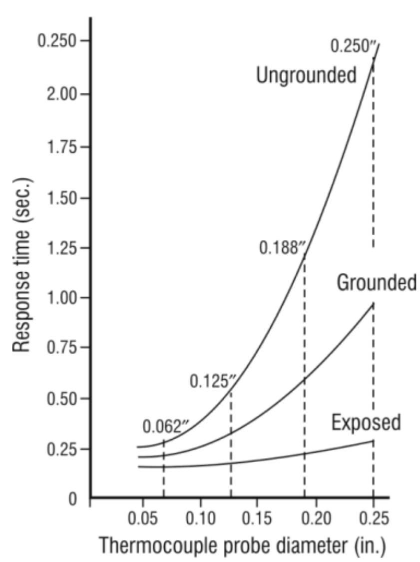
b. Which of the following relationships does the nonlinear graph appear to resemble:

1) $y = x^2$

2) $y = x$

3) $y = 1/x$

5. A technical handbook for temperature-measuring equipment shows the following graph of response times for various types of thermocouple constructions.



a. Identify the three types of thermocouple probes shown by the three curves in the graph.

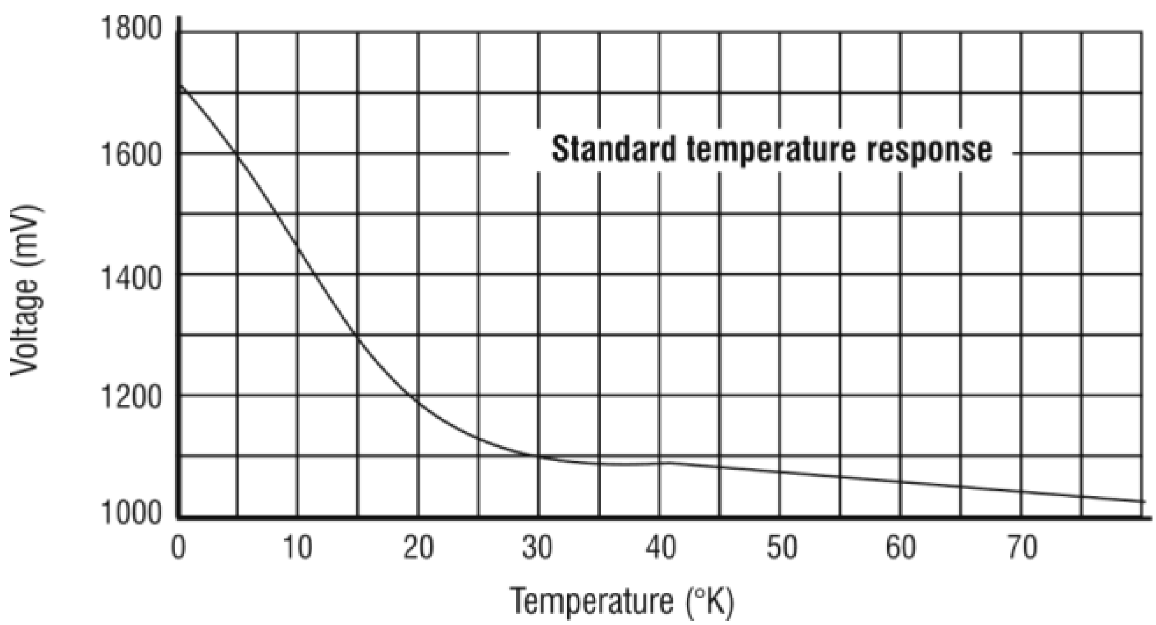
Nonlinear Equations Worksheet

b. Based on this graph, which type of thermocouple appears most sensitive to the diameter of the probe with regard to response time? Explain your answer.

c. If you were concerned about response time, and considering probes with a diameter of about 0.05 inch, would one type probe perform significantly better than another? What if you were considering probes with diameters of 0.25 inch?

d. Could you use a linear equation to approximate any of the regions of any of these curves? If so, describe those regions.

6. With the rising popularity of superconductors, measurement of very low temperatures has become critical. The temperature scale of Fahrenheit is no longer convenient, and so the Kelvin temperature scale is used. In the Kelvin scale, 0°K is called absolute zero (the lowest possible temperature), and the freezing point of water is at 273°K. Superconducting devices operate very close to 0°K. To measure these low temperatures requires special equipment. On the next page is a voltage response curve for one type of sensor.



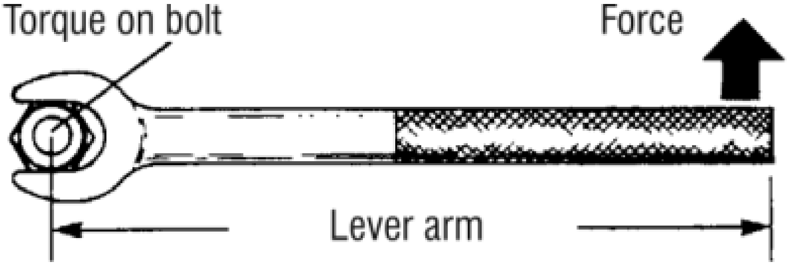
a. Identify two ranges of temperatures in the graph for which the response curve is very nearly linear.

b. For each range above, determine the approximate slope of the graph in the region. (Round to the nearest 1 mV per K°.)

c. What advantage is it for the slope to be so much greater in the region close to 0°K?

Nonlinear Equations Worksheet

7. The reference manuals for many automobiles specify how hard you should tighten bolts in various places on the car. This is done by specifying the ―torque‖ to apply to the bolt. The torque is equal to the force multiplied by the length of the lever arm, as the drawing shows.



You can compute the force needed when using wrenches of different lengths by rearranging the relationship to obtain the expression below.

$$\text{Force} = \frac{\text{Torque}}{\text{Lever arm}}$$

where *Torque* is measured in foot-pounds,
Lever arm is measured in feet, and
Force is measured in pounds.

a. For a torque of 45 foot-pounds, make a table and graph on your graph paper of the forces required when using lever arms ranging from 6 inches to 3 feet. List your axis appropriately (make a note that this is your graph for Problem 7).

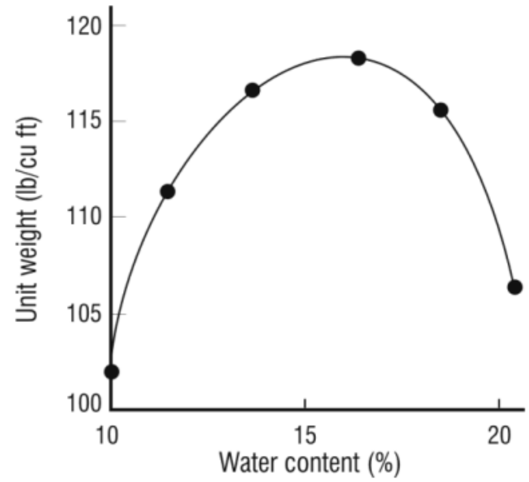
b. Is this relationship linear or nonlinear?

c. What is the difference in required force if you change from a small wrench that is 6 inches long to one that is 12 inches long? What difference is there when going from a 2-ft-long wrench to a 3-ft-long wrench?

Nonlinear Equations Worksheet

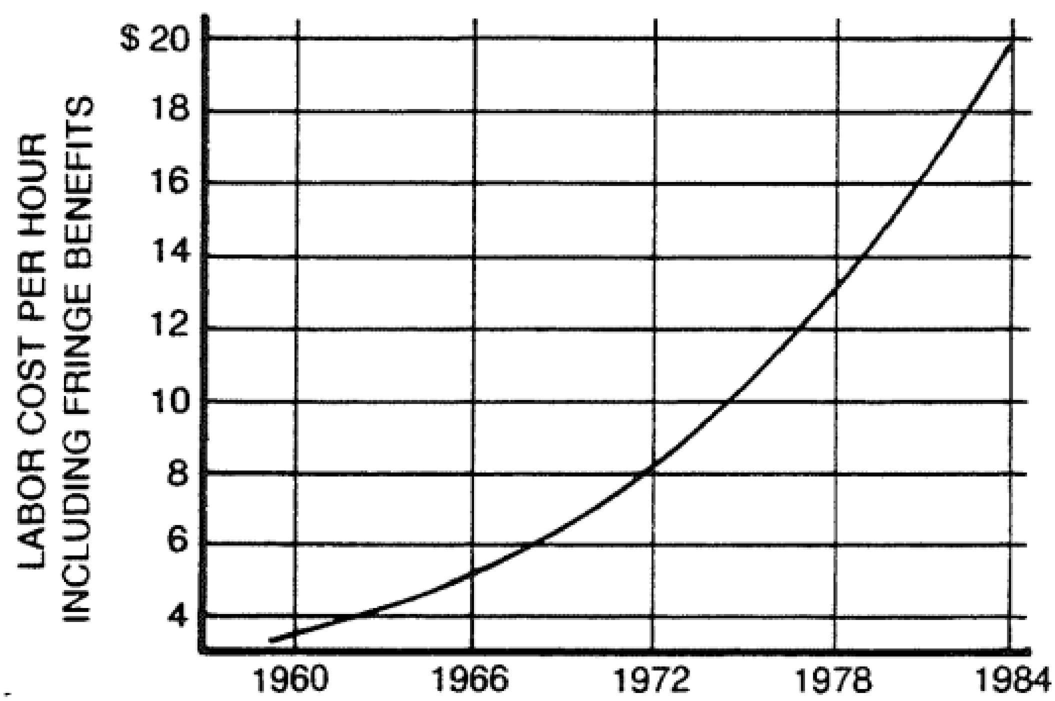
Name: ______________________ Date: ________________ Class: _____

1. The water content of soil affects its weight. The results of certain soil tests produce the results shown below as a graph (called a compaction curve).



   a. Examine the curve that is drawn through the test data. Does the trend it illustrates appear linear or nonlinear?
      The curve definitely appears nonlinear.
   b. The most important feature of the curve is the maximum, that is, the high point of the peak. The maximum weight shown on the graph is called the maximum dry density, or the M.D.D. The percent water content at which this maximum occurs is called the optimum moisture, or O.M. of the soil. Approximately what are the M.D.D and the O.M. of this soil sample?
      Answers between 118 and 119 lb/cu ft for the M.D.D.
      Answers between 16 and 17% for the O.M. should be considered acceptable

2. The graph below illustrates the changing labor costs in the American automotive industry over a 25-year period.



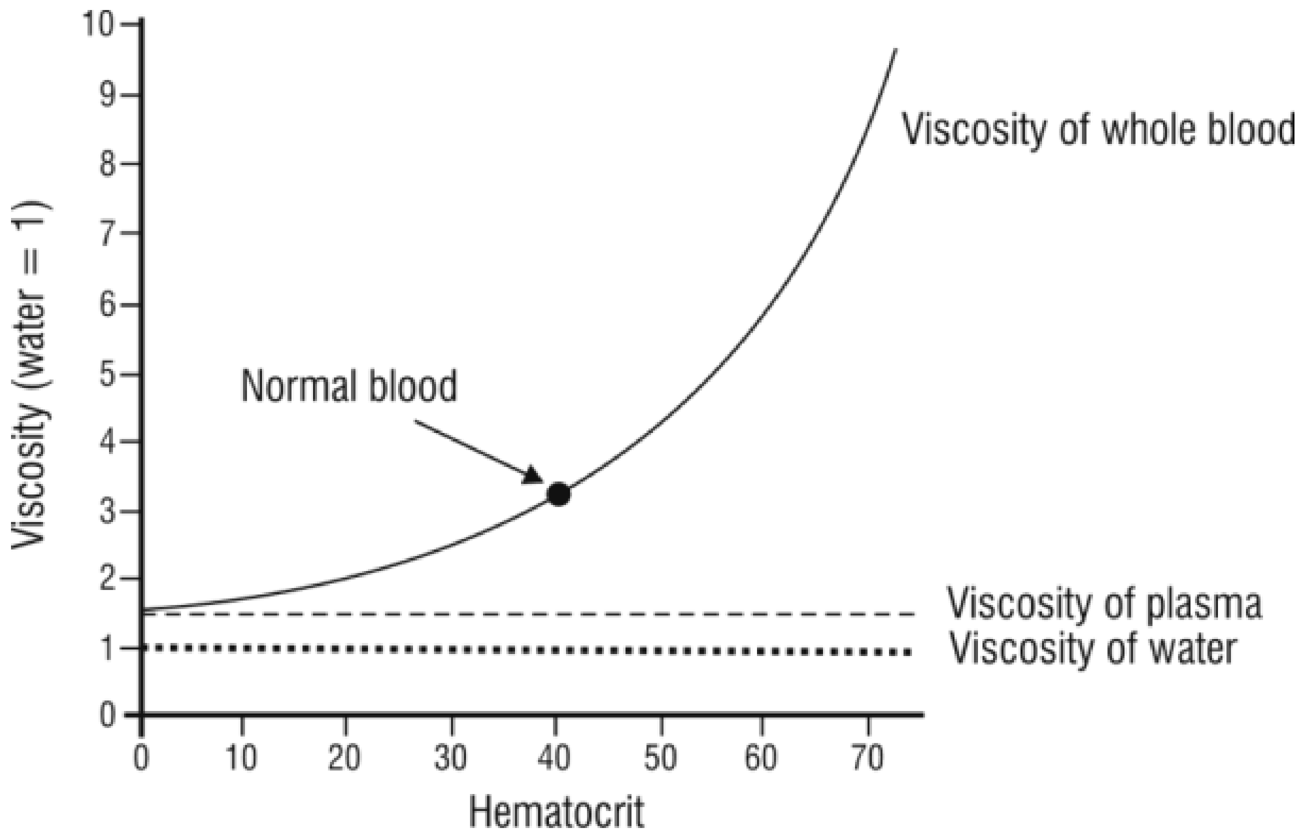
Nonlinear Equations Worksheet

a. Is this a graph of a linear trend or a nonlinear trend?
   The graph, since the line is curved, illustrates a nonlinear trend

b. The shape of the curve most resembles that of which of the following relationships:
   1) $y = x^2$
   2) $y = x$
   3) $y = 1/x$

c. Compute the slope of the small section of the graph for the period of the 1960s, and then also for the 1970s. (Round your answer to the nearest $0.01 per hour) Did you get about the same values? Explain.
   The students' answers may vary slightly, depending on the reading of the graph. The slope of the small section from 1962 to 1969 will serve to provide an average slope for the 1960s.
   Slope = $0.39 per year (rounded)
   For the 1970s, the span of 1972 through 1978 will suffice.
   Slope = $0.79 per year (rounded)
   The values of the slope for the two periods are NOT the same. The slope for the 1970s is almost twice that for the 1960s. This means that the automotive workers' pay during the 1970s was increasing almost twice as fast as it was during the 1960s.

3. When the count of red blood cells increases, there is more friction between the layers of cells. This friction determines the thickness, or viscosity, of the blood. A common measure of the blood-cell count is the hematocrit—the percentage of the blood by volume that is cells. A graph of the effect of various hematocrit values is shown below. (Note that the viscosity of water is shown as equal to 1 on the graph, for comparison with whole blood and with plasma.)



a. Which of the curves in the graph appears to be nonlinear?
   Viscosity of whole blood is definitely nonlinear, as evidenced by the curve.

Nonlinear Equations Worksheet

b. Which of the following relationships does the nonlinear graph appear to resemble:

1) $y = x^2$

2) $y = \sqrt{x}$

3) $y = {}^1/_x$

c. Consider the nonlinear curve for low values of hematocrit and for higher values. In which case does a small change in the hematocrit have more effect on the viscosity?

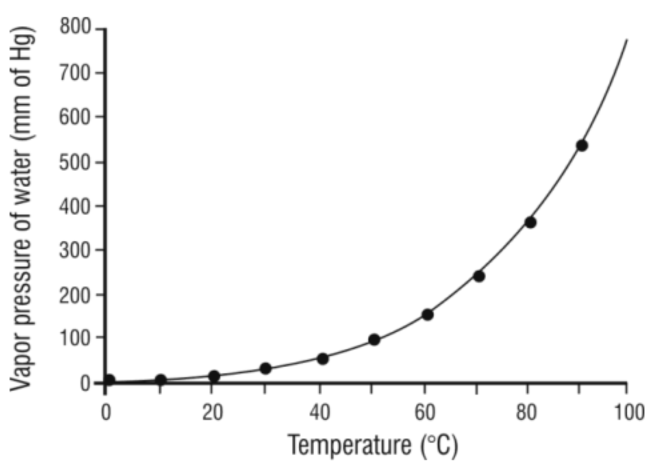
A small change in the hematocrit values has the most effect on viscosity when the hematocrit values are high. In other words, the slope is steeper for higher values of hematocrit.

4. The vapor pressure of water is very important when we consider breathing. A table of the vapor pressures of water (in units of millimeters of mercury) for various temperatures is shown below.

Temp (°C)	Pressure (mm Hg)
0	4.6
10	9.1
20	17.4
30	31.5
40	54.9
50	92.0
60	148.9
70	233.3
80	354.9
90	525.5
100	760.0

a. On your graph paper, draw a graph of the vapor pressure of water for the temperatures listed in the table above. List your axis appropriately (make a note that this is your graph for Problem 4).

The students' graphs should appear generally as shown below.



b. Which of the following relationships does the nonlinear graph appear to resemble:

1) $y = x^2$

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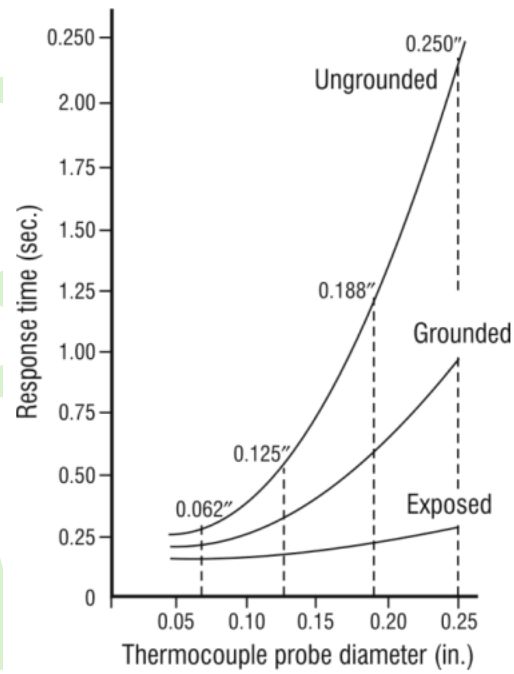
Nonlinear Equations Worksheet

5. A technical handbook for temperature-measuring equipment shows the following graph of response times for various types of thermocouple constructions.



   a. Identify the three types of thermocouple probes shown by the three curves in the graph.
   Ungrounded, grounded, and exposed
   b. Based on this graph, which type of thermocouple appears most sensitive to the diameter of the probe with regard to response time? Explain your answer.
   The ungrounded type of probe appears most sensitive, as evidenced by the steep slope of the graph. A slight increase in the ungrounded probe's diameter has a much more pronounced effect on the response time than the other types.
   c. If you were concerned about response time, and considering probes with a diameter of about 0.05 inch, would one type probe perform significantly better than another? What if you were considering probes with diameters of 0.25 inch?
   For probes of about 0.05″ diameter, all three types have essentially the same response time. But for larger probes of diameter 0.25″, the three probes have widely different response times. The exposed type of probe has a much faster response time than either of the other types, while the ungrounded type is, by far, the slowest of the three types.
   d. Could you use a linear equation to approximate any of the regions of any of these curves? If so, describe those regions.
   Those sections of the graphed curves that are reasonably straight could be approximated by linear equations. For example, the ungrounded curve, between diameters of 0.188 and 0.25 is essentially linear. Similarly, the curve for the exposed probes between diameters of about 0.125 and 0.25 is very straight, and could easily be approximated by a linear equation.

6. With the rising popularity of superconductors, measurement of very low temperatures has become critical. The temperature scale of Fahrenheit is no longer convenient, and so the Kelvin temperature scale is used. In the Kelvin scale, 0°K is called absolute zero (the lowest possible temperature), and the freezing point of water is at 273°K. Superconducting devices operate very close to 0°K. To measure these low temperatures requires special equipment. On the next page is a voltage response curve for one type of sensor.

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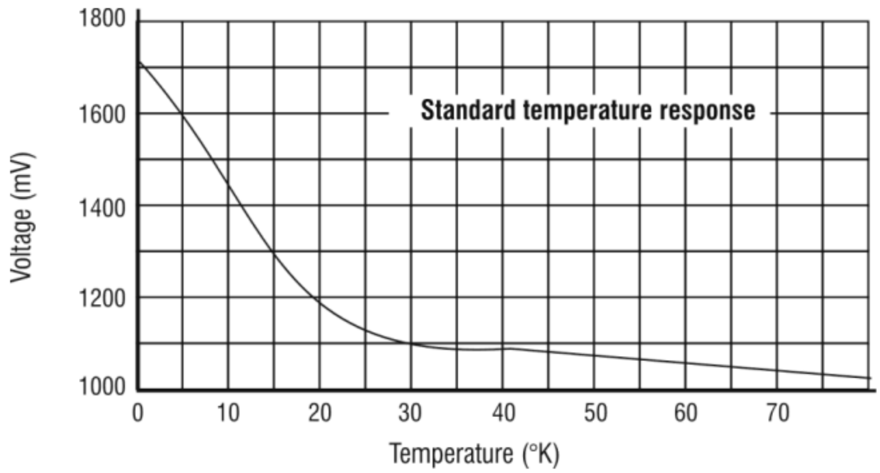
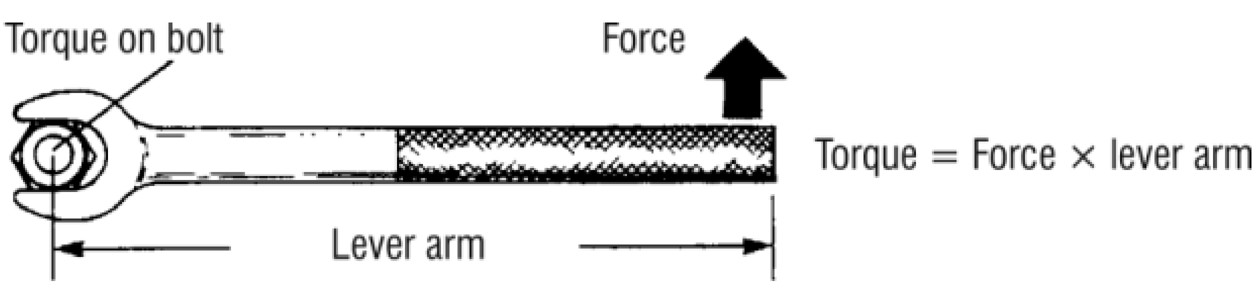


a. Identify two ranges of temperatures in the graph for which the response curve is very nearly linear.
The graph is very close to being linear in the range from 0 K to 20 K, and again in the range from 30 K to 80 K.

b. For each range above, determine the approximate slope of the graph in the region. (Round to the nearest 1 mV per K°.)
For the first range, from 0 K to 20 K, Slope = – 25 mV/K
For the second range, Slope = – 2 mV/K

c. What advantage is it for the slope to be so much greater in the region close to 0°K?
Since the slope is so much greater (over 10 times as great) in the lowtemperature region, the device is very sensitive to temperature changes in this region.

7. The reference manuals for many automobiles specify how hard you should tighten bolts in various places on the car. This is done by specifying the —torque‖ to apply to the bolt. The torque is equal to the force multiplied by the length of the lever arm, as the drawing shows.



You can compute the force needed when using wrenches of different lengths by rearranging the relationship to obtain the expression below.

$$\text{Force} = \frac{\text{Torque}}{\text{Lever arm}}$$

where *Torque* is measured in foot-pounds,
Lever arm is measured in feet, and
Force is measured in pounds.

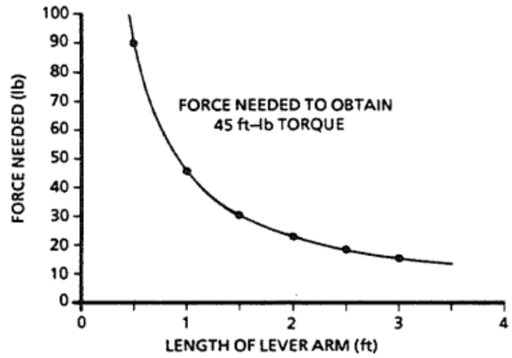
Nonlinear Equations Worksheet

a. For a torque of 45 foot-pounds, make a table and graph on your graph paper of the forces required when using lever arms ranging from 6 inches to 3 feet. List your axis appropriately (make a note that this is your graph for Problem 7).

The students' tables and graph should appear generally as shown below.

For a 45 ft-lb torque

Lever arm (ft)	Force (lb)
0.5	90
1.0	45
1.5	30
2.0	22.5
2.5	18
3.0	15



b. Is this relationship linear or nonlinear?

This relationship is nonlinear—an inverse relationship, characteristic of 1/x.

c. What is the difference in required force if you change from a small wrench that is 6 inches long to one that is 12 inches long? What difference is there when going from a 2-ft-long wrench to a 3-ft-long wrench?

From the table, the force needed is reduced by 45 pounds by changing from a 6-inch wrench to a 12-inch wrench. On the other hand, when changing from a 2-foot to a 3-foot wrench, the required force changes by only 7.5 pounds.

Nonlinear Equations Exit Ticket

Name: ______________________ Date: _________________ Class: _____

Inflation causes the prices of consumer goods to increase. For example, an average loaf of bread that costs $0.39 at one time might cost $1.05 ten years later. This is due mainly to inflation. The formula below relates the inflated prices of goods to a constant inflation percentage.

$$C = P(1 + i)^n$$

Where…

"C" is the current price after n years of constant inflation,

"P" is the price at the start of the time period,

"i" is the annual inflation rate, as a decimal value, and

"n" is the number of years.

1. Use the example bread prices given above for P and C, and the time span of ten years for n. Substitute these values into the equation above (Show your work below).

2. Isolate the variable i from your equation. (Hint: First isolate the factor that is raised to a power, then —undo‖ the power by taking the 10th root of both sides of the equation.) What was the average annual inflation rate i during this ten year span? (Show your answer with three significant digits.)

BONUS: Select another item for which you know the prices both now and several years ago. (You might ask your parents or friends, or check old newspaper records in a library.) Use the equation to estimate the average inflation rate during the time you selected.

Nonlinear Equations Exit Ticket

Name: ______________________ Date: _________________ Class: _____

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1. Use the example bread prices given above for P and C, and the time span of ten years for n. Substitute these values into the equation above (Show your work below).

$$1.05 = 0.39\,(\,1 + i\,)^{10}$$

2. Isolate the variable i from your equation. (Hint: First isolate the factor that is raised to a power, then —undo‖ the power by taking the 10th root of both sides of the equation.) What was the average annual inflation rate i during this ten year span? (Show your answer with three significant digits.)

To isolate the variable i, first divide both sides by 0.39…

$$\frac{1.05}{0.39} = \frac{0.39\,(1 + \mathrm{i})^{10}}{0.39}$$

Simplify and, as suggested in the text, take the 10th root of both sides.

$$\sqrt[10]{2.69} = \sqrt[10]{(1 + i)^{10}}$$

Simplify, and isolate i by subtracting 1 from both sides.

$$\sqrt[10]{2.69} - 1 = 1 + i - 1$$

$$i = \sqrt[10]{2.69} - 1$$

$i = 0.104$, or 10.4% annual inflation rate

Nonlinear Equations Exit Ticket

BONUS: Select another item for which you know the prices both now and several years ago. (You might ask your parents or friends, or check old newspaper records in a library.) Use the equation to estimate the average inflation rate during the time you selected.