WAMC Lab Template

Math Concept(s): Probability

Source / Text: Citizen Math (https://www.citizenmath.com/lessons/happy-meal)

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Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

Students will use dice and probability to discover how long it takes to get every toy in a Happy Meal set.

Lab Plan

Lab Title: Happy Meal Math

Prerequisite skills: Probability, mathematical deduction

Lab objective: Students will be able to:

- Create frequency distribution from experimental data and interpret the results
- Use theoretical and experimental probability to reason about real-world decisions
- Calculate expected value and use it to evaluate decisions in a real-world context

Standards: *(Note SPECIFIC relationship to Science, Technology, and/or Engineering)*

Mathematics K–12 Learning Standards:

- 7.SP.C: . Investigate chance processes and develop, use, and evaluate probability
- Models.
- HSS-ID.A.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).
- HSS.MD.B.7: Analyze decisions and strategies using probability concepts.

Standards for Mathematical Practice:

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- RI.4: Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings.
- W.4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Technology

- 3.d. Students build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness
☒ Financial/Economic/Business/Entrepreneurial Literacy
☐ Civic Literacy
☐ Health/Safety Literacy
☐ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

Creativity and Innovation
☒ Think Creatively
☒ Work Creatively with Others
☐ Implement Innovations

Critical Thinking and Problem Solving
☒ Reason Effectively
☒ Use Systems Thinking
☒ Make Judgments and Decisions
☒ Solve Problems

Communication and Collaboration
☒ Communicate Clearly
☐ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

Information Literacy
☐ Access and Evaluate Information
☒ Use and manage Information

Media Literacy
☐ Analyze Media
☐ Create Media Products

Information, Communications and Technology (ICT Literacy)
☒ Apply Technology Effectively

LIFE & CAREER SKILLS

Flexibility and Adaptability
☒ Adapt to Change
☒ Be Flexible

Initiative and Self-Direction
☒ Manage Goals and Time
☒ Work Independently
☒ Be Self-Directed Learners

Social and Cross-Cultural
☐ Interact Effectively with Others
☐ Work Effectively in Diverse Teams

Productivity and Accountability
☒ Manage Projects
☒ Produce Results

Leadership and Responsibility
☐ Guide and Lead Others
☐ Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Student hand outs
- Dice
- Calculators

Set-Up Required:

- Make sure every student has a handout and a die/number cube.

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

- Students will work individually or in pairs to run the experimental probability on how many Happy Meals it would take to receive all 6 toys. They will complete this experiment 4 times.
- Once they have completed the experiment 4 times, they will create a histogram of their findings, and calculate how much it would cost to get all the toys.
- They will then use their findings to calculate the cost of different scenarios and also discuss how purchasing the toys individually would work.

Cooperative Learning:

- Lead the class in a discussion after they have completed the first part of the worksheet to determine if they would want to attempt to get the toys in real life. Have students discuss with each other the pros and cons of random distribution of toys versus the restaurant planning out when to give them out.

Expectations:

- Students should be able to work independently or with a partner to complete the worksheet in one class period.
- The students should show their work, and be actively involved in the process of determining the probability.

Timeline:

- 1 - 45-50min class period

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab:

- Businesses deciding how to distribute items
- Probability used in weather, sports, online gaming

Career Applications:

- Insurance
- Business

Optional or Extension Activities:

- Students can develop business plans on how they could maximize their profit on Happy Meals

Real-World Takeaways

- When trying to collect a full set of Happy Meal toys, the chance of getting a *new* toy decreases as you collect more toys.
- The expected cost of acquiring a new toy, then, also increases.

Mathematical Objectives

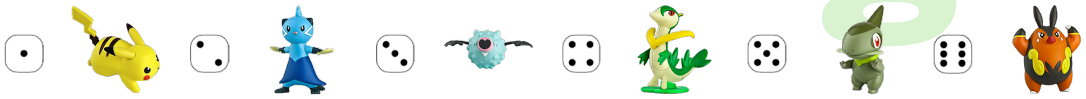
- Create frequency distribution from experimental data and interpret the results
- Use theoretical and experimental probability to reason about real-world decisions
- Calculate expected value and use it to evaluate decisions in a real-world context

Lesson at a Glance			120 min.
Launch		Watch a commercial about Happy Meals at McDonald's; share prior experiences trying to collect certain toys in them, and discuss how long it might take to get a whole collection.	5 min.
Part One	1	Use a die or random number generator to simulate buying Happy Meals until you've collected all six items in a set. Record the results from four trials.	15 min.
	2	Aggregate trials from the class to create a histogram showing how many times each number of rolls was required. Calculate the least and most someone would have to spend to get all six toys.	30 min.
Part Two	3	Calculate how many additional meals are *expected* to be needed to complete the set when one already has 5 toys, 4 toys, 3 toys, 2 toys, 1 toys.	40 min.
	4	Calculate how many Happy Meals you'd expect to buy to collect six different toys in a set, and then describe the effective cost per toy.	15 min.
	5	Discuss what a reasonable fee might be to let someone choose their Happy Meal toy, and debate whether or not that would be a good idea for McDonald's.	15 min.

Students watch the 1979 commercial announcing the all-new McDonald's Happy Meal for kids. According to the ad, each meal comes with a hamburger or cheeseburger, French fries, cookies, a soft drink...and a prize.

Discussion Questions
1. *Did you have Happy Meals when you were a kid? If so, what were some of your favorite prizes?* 2. *Today's Happy Meals often come with toys from TV shows and video games. How many toys do you expect are in a typical set? How many meals do you think someone would need to buy to get all the toys?* 3. *Why do you think McDonald's includes toys in Happy Meals?*
Key Takeaways
• Like they did in 1979, today's Happy Meals include toys. Each toy is typically part of a larger series, and customers who want to collect a complete set may have to buy more meals than there are toys.

1 In 1979, McDonald's introduced the Happy Meal. In addition to a sandwich, fries, and drink, the Happy Meal also included a small toy. Roll a die to model how many meals you'd need to get all six Pokémon figurines below. Repeat the experiment four times and record your results.



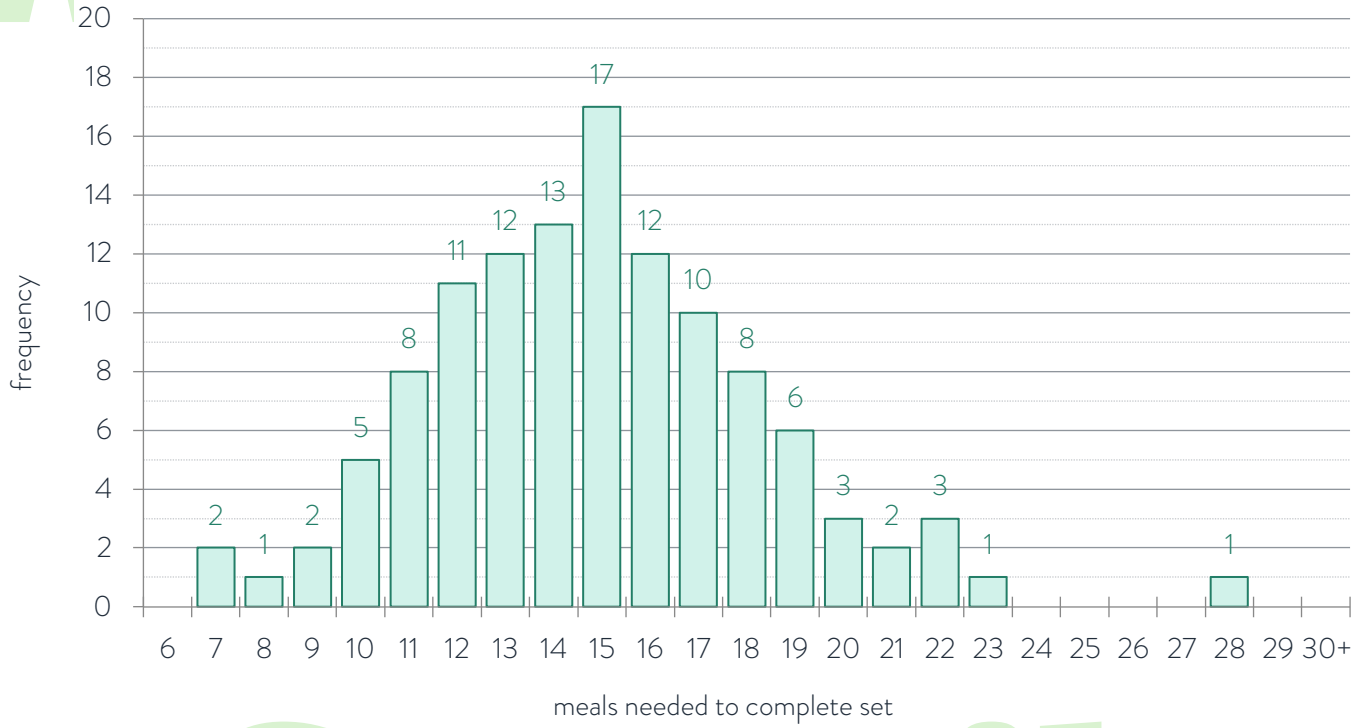
Pikachu	Dewott	Woobat	Servine	Axew	Pignite	Total Meals
|	|	| | |	| | |	| | | |	| | |	15
𝍸	| | | |	|	𝍸 |	|	| |	19
| |	|	| |	|	| |	|	9
| |	| |	| | |	| |	| | |	|	13

Results will vary.

Teacher Tip

To minimize the number of dice you need – and to make the activity more collaborative and fun – consider having students work in small groups. If you don't have dice, students can use a random number generator on their calculators; on a TI calculator, the input randInt(1,6) will provide a random integer from 1 to 6. Alternatively, students could draw numbers from a hat.

2 Gather the data from your class and create a histogram of the number of meals needed to collect all six toys. If a Happy Meal costs $3, what's the least anyone would have had to spend to get the complete set? The most?



Results will vary.

In the distribution above, the minimum number of meals that anyone needed to collect all the toys was 7. This occurred twice. In this scenario, it would have cost $3 • 7 = $21 to collect all the Pokémon toys. The greatest number of meals that anyone needed was 28. This happened once, and would have resulted in the complete set of toys costing $3 • 28 = $84.

The most common number of meals needed was 15 meals, which occurred 17 times. Here, the complete set would have cost $3 • $15 = $45.

3 Imagine you already have five Pokémon toys. Assuming you're equally likely to get each toy, how many *more* Happy Meals would you expect to have to buy to complete the set? What if you only have four toys? Two toys?

	Meals Needed to Get a New Unique Toy						Expected Add'l Meals Needed
	1st Toy	2nd Toy	3rd Toy	4th Toy	5th Toy	6th Toy	
Scenario A						p(new toy) = 1/6 6 meals	6 more Happy Meals
Scenario B					p(new toy) = 2/6 = 1/3 3 meals	p(new toy) = 1/6 6 meals	9 more Happy Meals
Scenario C			p(new toy) = 4/6 = 2/3 1.5 meals	p(new toy) = 3/6 = 1/2 2 meals	p(new toy) = 2/6 = 1/3 3 meals	p(new toy) = 1/6 6 meals	12.5 more Happy Meals

If someone already has 5 toys (Scenario A), then each additional meal will have a 1 in 6 chance of including the toy they want (successful) and a 5 in 6 chance of including a toy they already have (unsuccessful). Therefore they'd expect to need six additional meals to complete the set. It's possible that they'd only need one. It's also possible that they'd need 100. But on average, they'd need six.

If someone already has 4 toys (Scenario B), the next meal has a 2 in 6 chance of being successful, and they'd expect to have to buy an additional 6/2 = 3 meals to get a new toy. If they already have 2 toys (Scenario C), the next meal has a 3 in 6 chance of being successful, in which case I'd expect to need 2 Happy Meals to get a toy I don't already have. Whatever the success fraction is for a given number of toys, the number of additional meals someone would expect to need is the reciprocal. If a meal has a k/6 chance of being successful, then it'll require on average 6/k additional meals to get a toy the person doesn't already have.

Teacher Tip

Some students may find it intuitive that if a meal has a 1/6 chance of providing a new toy, then on average they'd need to buy 6 meals to complete the set. Demonstrating why this is true involves expected value and limits. If someone already has five toys, the expected value of m Happy Meals is:

$$\text{Expected value of m meals if you already have 5 toys} = 1\left(\frac{1}{6}\right) + 2\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + 3\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \ldots = \sum_{n=1}^{m} n\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)$$

You can think of the equation as, "[1 meal • probability I succeed on the first meal] + [2 meals • probability I succeed on the second meal] + [3 meals • probability I succeed on the third meal] + ..." When you graph the equation, you see that it has a horizontal asymptote at 6, i.e. the limit as $m \to \infty$ is 6. In other words, you expect to need six meals to get the missing toy. More generally, if you already have t toys, each meal has a (6 – t)/6 chance of success, and the limit is 6/(6 – t) as $m \to \infty$.

$$\text{Expected value of m meals if you already have t toys} = \sum_{n=1}^{m} n\left(\frac{t}{6}\right)^{n-1}\left(\frac{6-t}{6}\right)$$

4 Starting from scratch, calculate the number of Happy Meals you'd expect to buy to get all six Pokémon figurines. Based on this, how much would you say that each toy really costs?

Meal 1	Meal 2	Meal 3	Meal 4	Meal 5	Meal 6
no toys, so new toy is guaranteed	chance of new toy = 5/6	chance of new toy = 4/6 = 2/3	chance of new toy = 3/6 = 1/2	chance of new toy = 2/6 = 1/3	chance of new toy = 1/6
1 meal	1.2 meals	1.5 meals	2 meals	3 meals	6 meals

Starting from scratch, someone would expect to buy 1 + 1.2 + 1.5 + 2 + 3 + 6 = 14.7 Happy Meals before getting a complete set of figurines. If each meal costs $3, they'd expect to spend a total of (14.7 meals • $3/meal ≈) $44 in total to get all six toys. Ignoring the food – in other words, assuming all the person really cares about is the toy – this implies a cost of ($44 ÷ 6 toys ≈) $7.33 per toy.

In reality, though, not all six toys cost the same. If a customer doesn't already have a toy – i.e. if they haven't bought any Happy Meals yet – they're guaranteed to get a new toy with their first meal. This implies that the first toy effectively costs $3. If a customer already have 5 toys, they'd expect to need six additional Happy Meals to complete the set, which implies that the final toy effectively costs $18. The more toys someone already has, the more expensive it is to get a toy they don't have. But on average, each toy costs $7.33.

Teacher Tip

Consider having students come up with a method for finding the expected number of Happy Meals needed to complete a set with any number of toys. For a collection with K toys, the expected number of meals needed to complete the set is:

$$\frac{K}{K-0} + \frac{K}{K-1} + \frac{K}{K-2} + \ldots \frac{K}{K-(K-1)} = \sum_{n=0}^{K-1} \frac{K}{K-n} = \sum_{n=1}^{K} \frac{K}{n}$$

5 McDonald's regularly updates its Happy Meal collections. Some have fewer than six toys, while others have more. In additional to distributing toys at random, imagine McDonald's offered a new option: for an additional fee, customers could specify which figurine they got. Do you think this would be a good idea? Why or why not

Customer's Perspective

- The more toys there are in a collection, the more meals a customer has to buy to complete the set. As the size of the set increases, the expected meals needed goes up by more and more (i.e. it's increasing at a non-linear rate). If customer cares mostly about collecting the toys, paying a fee could end up saving money (depending on what the fee is, of course). On the other hand, if the customer goes to McDonald's frequently already, paying a fee might be a waste of money.
- For parents who don't like fast food, a pay-to-choose option would minimize trips to McDonald's.
- If customers could choose their own figurines, then collecting a whole set might feel like less of an accomplishment

McDonald's Perspective

- When parents buy Happy Meals for their kids, they might also buy food for themselves. If a pay-to-choose option resulted in families coming to McDonald's less often, the restaurant might lose money on the parents' meals.
- Charging a fee might cause parents to view Happy Meals as being more about the toy than the food, especially if the fee were more than the price of a Happy Meal. In this case, parents might choose to buy figurines from an actual toy store instead of from McDonald's.

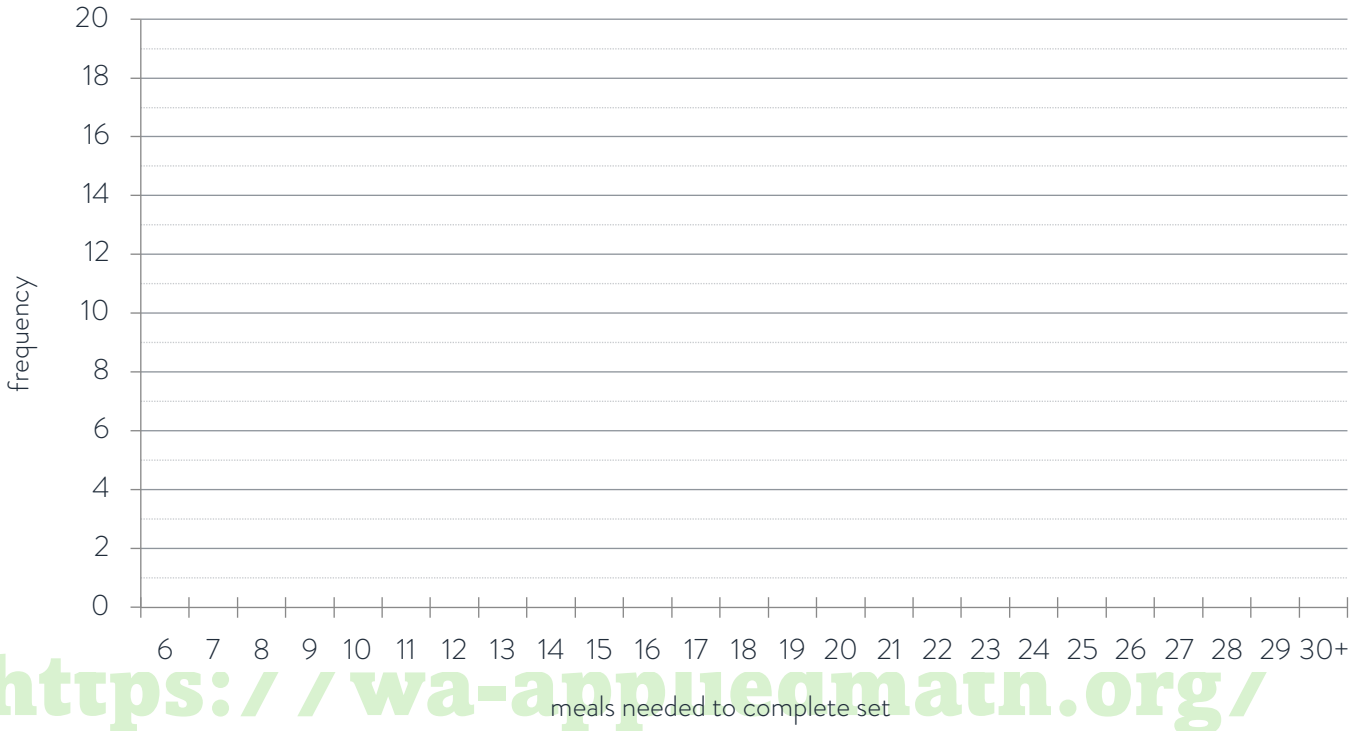
Name & Date:

Part One: Pay to Play

1 In 1979, McDonald's introduced the Happy Meal. In addition to a sandwich, fries, and drink, the Happy Meal also included a small toy. Roll a die to model how many meals you'd need to get all six Pokémon figurines below. Repeat the experiment four times and record your results.

Pikachu	Dewott	Woobat	Servine	Axew	Pignite	Total Meals

2 Gather the data from your class and create a histogram of the number of meals needed to collect all six toys. If a Happy Meal costs $3, what's the least anyone would have had to spend to get the complete set? The most?



Part Two: Pay [Not] to Play

3 Imagine you already have five Pokémon toys. Assuming you're equally likely to get each toy, how many *more* Happy Meals would you expect to have to buy to complete the set? What if you only have four toys? Two toys?

	Meals Needed to Get a New Unique Toy						Expected Add'l Meals Needed
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Scenario A							
Scenario B							
Scenario C							

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