WAMC Lab Template

Math Concept(s): Trigonometric Functions, Unit Circle, Radians
Source / Text: curriculum.illustrativemathematics.org
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Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

Lab Plan

Lab Title: Discover the Unit Circle

Prerequisite skills: Basic Right Triangle Trigonometry, Formula for the Circumference of a Circle
This lesson builds on the geometry course in which students learned that all circles are similar and examined arcs intercepted by given angles. That work led to defining the radian measure of an angle as the ratio of the arc length traveled to the radius of the circle. This means that 1 radian is the angle when the length of the arc it intersects on a circle of radius is. Students also learned that by this definition, and because is the ratio of the circumference of the circle to its diameter, there are radians in a full circle. This lesson includes an optional activity if students need practice recalling the definition of radian measurement.

Lab objective: This will be an initial investigation into this topic and will occur before instruction. The goal of this lesson is for students to begin their exploration of the unit circle, defined as a circle of radius 1 centered at the origin, which they continue in the following lesson and use throughout the remainder of the unit. They focus first on the symmetric nature of the coordinates of points (x, y) on the unit circle and then learn that these points can also be defined by their angle of rotation, which leads to working with radian angle measurements.

Standards: *(Note SPECIFIC relationship to Science, Technology, and/or Engineering)*

Mathematics K–12 Learning Standards:

- Trigonometric Functions HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Standards for Mathematical Practice:

- CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.
- CCSS.MATH.PRACTICE.MP4 Model with mathematics.

K-12 Learning Standards-ELA (Reading, Writing, Speaking & Listening):

- CCSS.ELA-LITERACY.RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

K-12 Science Standards

- MS-ESS2-3 Patterns in rates of change and other numerical relationships can provide information about natural systems.

Technology

- 5-ESS1-2. Represent data in graphical displays (bar graphs, pictograph and/or pie charts) to reveal patterns that indicate relationships.

Engineering

- 4-PS4-3. Generate and compare multiple solutions that use patterns to transfer information.

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

☐ Global Awareness ☐ Financial/Economic/Business/Entrepreneurial Literacy ☐ Civic Literacy
☒ Health/Safety Literacy ☐ Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION
Creativity and Innovation
☒ Think Creatively
☒ Work Creatively with Others
☐ Implement Innovations
Critical Thinking and Problem Solving
☒ Reason Effectively
☒ Use Systems Thinking
☐ Make Judgments and Decisions
☒ Solve Problems
Communication and Collaboration
☒ Communicate Clearly
☒ Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS
Information Literacy
☒ Access and Evaluate Information
☒ Use and manage Information
Media Literacy
☐ Analyze Media
☐ Create Media Products
Information, Communications and Technology (ICT Literacy)
☒ Apply Technology Effectively

LIFE & CAREER SKILLS
Flexibility and Adaptability
☐ Adapt to Change
☒ Be Flexible
Initiative and Self-Direction
☐ Manage Goals and Time
☒ Work Independently
☒ Be Self-Directed Learners
Social and Cross-Cultural
☒ Interact Effectively with Others
☐ Work Effectively in Diverse Teams

Productivity and Accountability
☐ Manage Projects
☒ Produce Results
Leadership and Responsibility
☒ Guide and Lead Others
☒Be Responsible to Others

Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Various Round Objects for Students to Use
- String
- Rulers

Set-Up Required:

- Arrange Round Object around the room in stations for student teams to visit
- Cut pieces of string for each team

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

- Students must collaborate with their team and discuss outcomes and identify patterns
- Students will fulfill different roles throughout the lab (materials, secretary/scribe, measurement, communication)

Cooperative Learning:

- Students must work with their team to divide up the tasks, so everyone gets a chance to fulfill each role (secretary/scribe, measurements, communications, materials)
- Everyone must be heard and all group members questions should be addressed

Expectations:

- Students will measure each radius and cut string to reflect the radius' length
- Students will measure the circumference of the circle and express that circumference in how many radii the circumference equals (They will estimate to the best of their ability with the help of their team)
- Students will identify patterns as they measure the different size round objects at each station
- Students will complete a data sheet and answer all questions on the lab worksheet.

Timeline:

- One 2-hour class period

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- Students will be able to use the concept of circumference, radius and the unit circle to understand the unit circle which they can use to solve various problems involving right triangle trigonometry (angles of elevation, angles of depression, lengths of ladders needed to reach various locations on jobs, etc.)

Career Applications

- This lab will identify parts of the unit circle that can be used to solve problems in engineering, surveying, map-making, computer graphics and many others.

Optional or Extension Activities

- Students look for regularity in repeated reasoning as they apply radian measure to examine the distance a wheel travels as it rolls for several angles, reasoning that the measure of the angle of revolution corresponds to the distance traveled when the radius is 1 (MP8).

Lab Instructions: Discover the Unit Circle

Your team needs to measure the radius of each round object around the room. Then cut the string to reflect the length of the radius. Use this piece of string to measure the circumference. Express the circumference in terms of how many radii it measures (estimate to the nearest 10^{th}.)

1. What pattern do you see after measureing all of the round objects around the room?
2. About how many radii does it take to go halfway around the circle?
3. About how many radii does it take to go all the way around the circle?
4. Compare your answers to the previous two questions with your partners.
5. What is the exact number of radii that fit around the circumference of the circle? Explain how you know.
6. Why doesn't the number of radii that fit around the circumference of a circle depend on the radius of the circle? Explain how you know.

Group Discussion…Be prepared to share your findings.

*Teacher will make sure everyone understands how this activity relates to the Circumference Formula and how this leads to measuring angles in radians rather than degrees.

The purpose of this discussion is to ensure that all students understand why the number of radii that fit around the circumference of a circle does not depend on the size of the radius of the circle. Begin the discussion by inviting students to share how many radii it took them to go all the way around a circle, recording responses for all to see.

Invite previously identified students to share their reasoning about the exact number of radii that fit around the circumference of a circle. If not mentioned, make sure students recall that the circumference of a circle is proportional to the diameter, d and that the constant of proportionality is π, which we can see in the equation $C = 2\pi r$ or $C = \pi d$.

Next, select students to share their explanation for why the number of radii that fit around the circumference of a circle doesn't depend on the radius of the circle. Record student explanations and any diagrams used for all to see.

Rubric

Discover the Unit Circle	Points
Students correctly measured the radius and circumference and estimated reasonably.	0- not correct 1- minor mistakes 2- correct measurments and estimation of circumference in terms of radii
Students express thoroghly how they know how many radii the circumference measures and relate that to the formula for circumference.	0- no progress 1- partial correct measurements 2- correct measurements using the radius string 3- correct measurements using the radius string and correct estimate 4- correct measurements using the radius string, correct estimate and correctly relates it to the formula for Circumference 5- correct measurements using the radius string, correct estimate, correctly relates it to the formula for circumference and throughly correct explanations with examples
Mathematical Practices: Construct viable arguments and critique the reasoning of others furing the discussuion. Students should include logical and thorough reasoning.	3- For demonstration of practice. Partial credit can be awarded.

Total Points: 10 points

WAMC Lesson Plan

Name(s): Victoria Kelley
Email Address: Kelley@skschools.org
Lesson Title: The Unit Circle Explained
Date: 06/21/2022
Text: Big Ideas – Algebra 2 STEM Correlation: Math Lesson Length: 2 hrs.

Big Idea (Cluster): Students will understand the Unit Circle - Trigonometry

Mathematics K–12 Learning Standards: HSF-TF.A.2, Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Mathematical Practice(s): 4, 8

Content Objectives: Within 3 minutes, students will be able to correctly label all radian and degree measures as well as x & y coordinates of points on the circle corresponding to the 30, 60 & 90 degree reference angles from 0-360 degrees.

- **Language Objectives (ELL):** CCSS.ELA-LITERACY.RST.11-12.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Vocabulary:
Radian, Unit Circle, Reference Angle, Quadrant, Coordinates

Connections to Prior Learning:
Right Triangle Trigonometry

Questions to Develop Mathematical Thinking:
- What patterns do you see between the signs of the coordinates in each quadrant?
- What pattern do you see between the coordinates of each point and the long and short sides of the corresponding right triangles?
- How does the Pythagorean Theorem relate to the coordinates of each point?
- When using the Pythagorean Theorem, what is the length of the hypothenuse of each right triangle in the unit circle?

Common Misconceptions:
- Sinθ and Cosθ order in the ordered pairs (Cosθ, Sinθ)
- Evaluating Square Roots
- Answering in Degrees when Radians are called for and vice versa
- Not understanding how to evaluate expressions like Sin(π/6), thinking you want them to tell you it's 30 degrees rather than the side length, y-coordinate of the point
- Not understanding when calculating tangent, it's a ratio of the 2 coordinates.
- Not knowing how to reduce or rationalize denominators when calculating the tangent

Assessment (Formative and Summative):
- Students will fill out all angles (degrees and radians) and all coordinates of common angles

Materials:
- Blank Unit Circles (1 with 30-60-90 triangles, 1 with 45-45-90 degree triangles)

WAMC Lesson Plan

Instruction Plan:

Introduction: We will review the Discovering the Unit Circle lab findings
Explore: Students will calculate all values in quadrants II, III & IV after we do quadrant 1 together
When I observe students: they will be using the 30-60-90 and 45-45-90 triangle models to find all the coordinates of the points around the Unit Circle and noticing patterns. They will also determine all radian measures noticing 30-60-90 degree angles cut the circle into 12-1/6ths and the 45-45-90 degree angles cut the circle into 8-1/4ths.
Questions to Develop Mathematical Thinking as you observe: What patterns do you recognize? What do you notice about the signs in each quadrant?
Answers: The same values repeat depending on the short and long sides of the triangles. The signs follow those for points in each quadrant.
Summarize: Students will realize that the same values repeat around the circle from quadrant I with only the signs changes according to each quadrant.

Career Application(s):

- Engineering, geography, building, construction, sciences, etc.

Leadership/21st Century Skills:

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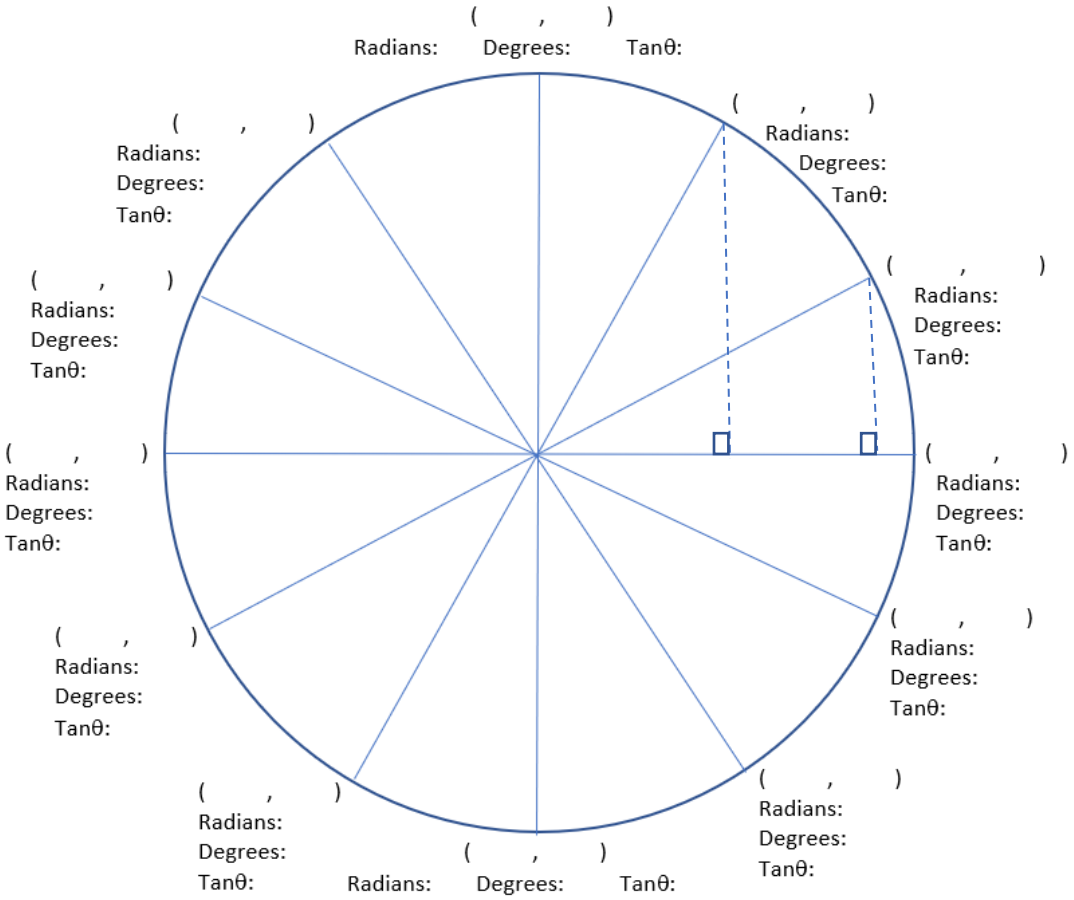
30° Angles Name:_____



45° Angles