

WAMC Lab Template

Math Concept(s): Law of Cosines, use of tools (rangefinder, compass and scientific calculator), oblique triangles (SAS), algebra (radicals) and trigonometric formula use.

Source / Text: None

Developed by: Chris Aiken E-Mail: caiken@eagles.edu Date: 6/21/22

Attach the following documents:

- Lab Instructions
- Student Handout(s)
- Rubric and/or Assessment Tool

Short Description (Be sure to include where in your instruction this lab takes place):

Students will go to the football field and use a variety of tools and techniques to calculate the distance from one student to the next using a rangefinder, a compass and the Law of Cosines (which includes the use of a calculator). Students will be broken into groups of 3 (students decide each role: Fire tower 1, Fire tower 2 and the fire). They will randomly space themselves out by roles. Once they are space out:

- Fire tower 1 will measure the distance from him/her to both the fire and the other fire tower (range finder). He/she will then use a compass to articulate the angle between the fire and the other fire tower. Once this is complete, he/she will diagram their oblique triangle, label the triangle and use the Law of Cosine to calculate the missing distance from the fire to the other fire tower.
- Fire tower 2 will measure the distance from him/her to both the fire and the other fire tower (range finder). He/she will then use a compass to articulate the angle between the fire and the other fire tower. Once this is complete, he/she will diagram their oblique triangle, label the triangle and use the Law of Cosine to calculate the missing distance from the fire to the other fire tower.
- The fire will measure the distance from him/her to both the fire towers (range finder). He/she will then use a compass to articulate the angle between the fire towers. Once this is complete, he/she will diagram their oblique triangle, label the triangle and use the Law of Cosine to calculate the missing distance between the fire towers.
- Lastly, the students will identify their angles to their teammates and add them up. If they have done this correctly, the angles will add up to 180° . If it doesn't, this is a great time to discuss the power/disservice of estimation and how that could affect their findings and how estimation affected their answers.

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Lab Plan

Lab Title: Where is the fire?

Students will be broken into groups of 3 (students decide each role: Fire tower 1, Fire tower 2 and the fire). They will randomly space themselves out on the football field by roles. Once they are space out:

This lesson will be conducted one group at a time. The other groups will watch and participate

- Fire tower 1 will measure the distance from him/her to both the fire and the other fire tower (range finder). He/she will then use a compass to articulate the angle between the fire and the other fire tower. Once this is complete, he/she will diagram their oblique triangle, label the triangle and use the Law of Cosine to calculate the missing distance from the fire to the other fire tower.
- Fire tower 2 will measure the distance from him/her to both the fire and the other fire tower (range finder). He/she will then use a compass to articulate the angle between the fire and the other fire tower. Once this is complete, he/she will diagram their oblique triangle, label the triangle and use the Law of Cosine to calculate the missing distance from the fire to the other fire tower.
- The fire will measure the distance from him/her to both the fire towers (range finder). He/she will then use a compass to articulate the angle between the fire towers. Once this is complete, he/she will diagram their oblique triangle, label the triangle and use the Law of Cosine to calculate the missing distance between the fire towers.
- Lastly, the students will identify their angles to their teammates and add them up. If they have done this correctly, the angles will add up to 180° . If it doesn't, this is a great time to discuss the power/disservice of estimation and how that could affect their findings and how estimation affected their answers.

Prerequisite skills:

- Students need to be able to use a rangefinder.
- Students need to be able to read a tape measure.
- Students need to be able to solve a basic linear equation (solve for x).
- Students need to be able to create a SAS triangle and label it.
- Students need to be able to use the Law of Cosines.

Lab objective:

- To engage students
- To create a scenario where students will use basic tools to establish information not already established.
- To create collaboration between 3 students.
- To create a scenario where students need to record data.
- To create a scenario where students use collected data in order to calculate a real life problem mathematically (both using algebra, geometry and the Law of Cosines).

- To create a scenario where students can discuss the differences between a perfect answer and an estimated value and when each can or should play a role in real life.
- To connect a mathematical problem to real life (specifically to the practice of forestry).

Standards: *(Note SPECIFIC relationship to Science, Technology, and/or Engineering)*

Mathematics K–12 Learning Standards:

- HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Technology

- 1.c. Students use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

Engineering

- HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Standards for Mathematical Practice:

- CCSS.MATH.CONTENT.HSG.SRT.D.10
(+) Prove the Laws of Sines and Cosines and use them to solve problems.
- CCSS.MATH.CONTENT.HSG.SRT.D.11
(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Leadership/21st Century Skills:

<u>21st Century Interdisciplinary themes</u> (Check those that apply to the above activity.)			
<input type="checkbox"/> Global Awareness	<input type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy	<input type="checkbox"/> Civic Literacy	
<input type="checkbox"/> Health/Safety Literacy	<input checked="" type="checkbox"/> Environmental Literacy		
<u>21st Century Skills</u> (Check those that students will demonstrate in the above activity.)			
LEARNING AND INNOVATION	INFORMATION, MEDIA & TECHNOLOGY SKILLS	LIFE & CAREER SKILLS	Productivity and Accountability
<u>Creativity and Innovation</u>	<u>Information Literacy</u>	<u>Flexibility and Adaptability</u>	<u>Accountability</u>
<input checked="" type="checkbox"/> Think Creatively	<input type="checkbox"/> Access and Evaluate Information	<input type="checkbox"/> Adapt to Change	<input type="checkbox"/> Manage Projects
<input checked="" type="checkbox"/> Work Creatively with Others	<input checked="" type="checkbox"/> Use and manage Information	<input checked="" type="checkbox"/> Be Flexible	<input checked="" type="checkbox"/> Produce Results
<input checked="" type="checkbox"/> Implement Innovations	<u>Media Literacy</u>	<u>Initiative and Self-Direction</u>	<u>Leadership and Responsibility</u>
<u>Critical Thinking and Problem Solving</u>	<input type="checkbox"/> Analyze Media	<input type="checkbox"/> Manage Goals and Time	<input type="checkbox"/> Guide and Lead Others
<input type="checkbox"/> Reason Effectively	<input type="checkbox"/> Create Media Products	<input type="checkbox"/> Work Independently	<input type="checkbox"/> Be Responsible to Others
<input type="checkbox"/> Use Systems Thinking	<u>Information, Communications and Technology (ICT Literacy)</u>	<u>Social and Cross-Cultural</u>	<input checked="" type="checkbox"/> Be Responsible to Others
<input checked="" type="checkbox"/> Make Judgments and Decisions	<input checked="" type="checkbox"/> Apply Technology Effectively	<input checked="" type="checkbox"/> Interact Effectively with Others	
<input checked="" type="checkbox"/> Solve Problems		<input type="checkbox"/> Work Effectively in Diverse Teams	
<u>Communication and Collaboration</u>			
<input checked="" type="checkbox"/> Communicate Clearly			
<input checked="" type="checkbox"/> Collaborate with Others			

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Teacher Preparation: (What materials and set-up are required for this lab?)

Materials

- Rangefinder
- Compass
- Calculator

Set-Up Required:

- Hand out the lab worksheet and go to the football field. The rangefinder and compasses will be transported to the field by myself.

Lab Organization Strategies:

Leadership (Connect to 21st Century Skills selected):

<u>21st Century Interdisciplinary themes</u> (Check those that apply to the above activity.)			
<input type="checkbox"/> Global Awareness	<input type="checkbox"/> Financial/Economic/Business/Entrepreneurial Literacy	<input type="checkbox"/> Civic Literacy	
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LEARNING AND INNOVATION	INFORMATION, MEDIA & TECHNOLOGY SKILLS	LIFE & CAREER SKILLS	Productivity and Accountability
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<input checked="" type="checkbox"/> Implement Innovations	<u>Media Literacy</u>	<u>Initiative and Self-Direction</u>	<input type="checkbox"/> Guide and Lead Others
<u>Critical Thinking and Problem Solving</u>	<input type="checkbox"/> Analyze Media	<input type="checkbox"/> Manage Goals and Time	<input type="checkbox"/> Work Independently
<input type="checkbox"/> Reason Effectively	<input type="checkbox"/> Create Media Products	<input type="checkbox"/> Be Self-Directed Learners	<input type="checkbox"/> Be Responsible to Others
<input type="checkbox"/> Use Systems Thinking	<u>Information, Communications and Technology (ICT Literacy)</u>	<u>Social and Cross-Cultural</u>	<input checked="" type="checkbox"/> Be Responsible to Others
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<u>Communication and Collaboration</u>			
<input checked="" type="checkbox"/> Communicate Clearly			
<input checked="" type="checkbox"/> Collaborate with Others			

Cooperative Learning:

- Students will work in groups of 3 (establishing and documenting their own roles).

Expectations:

- Students will assign themselves roles (fire tower 1, fire tower 2 and the fire)
- Students will collaborate
- Students will be respectful of each other

Timeline:

- 1 class period

Post Lab Follow-Up/Conclusions:

Discuss real world application of learning from lab

- This will occur the following class period to discuss the problems/benefits of this lab as it pertains to the real world. Their reflection sheet will be used to direct this process.

Career Applications

- Navigation
- Forestry
- Landscape architecture

Optional or Extension Activities

- None

WAMC Lesson Plan

Email Address: caiken@gmail.com

Lesson Title: Law of Cosines

Date:6/22/22

Text: None STEM Correlation: Real life application to mathematics Lesson Length: 55 min.

Big Idea (Cluster): Similar triangles and Proportionality

Mathematics K–12 Learning Standards:

K-12 Science Standards

- HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Technology

- 1.c. Students use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

Engineering

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(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Mathematical Practice(s):

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Model with mathematics
4. Use appropriate tools strategically
5. Attend to precision

Content Objectives: Students will be able to:

- Identify triangles by ASA, AAS, SAS and SSS.
- Understand that ASA and SSS triangles are suitable for the Law of Cosines.
- Be able to correlate the Law of Cosine formulas as they apply the ASA and SSS triangles by labeling the triangles correctly.
- Be able to calculate missing side lengths/angles for ASA and SSS triangles by use of the Law of Cosines.

Language Objectives (ELL):

Students will be able to verbally and productively collaborate, discuss and debate with each other about their mathematical findings.

WAMC Lesson Plan

Vocabulary: ASA, SSS, Law of Cosines, Law of Sines, Oblique triangles and Corresponding sides and angles.	Connections to Prior Learning <ul style="list-style-type: none">• Students have already learned and used Sine and Cosine with right triangles.• Students have already learned how to use sine and cosine to find missing sides and angles with right triangles.• Students have already learned SAS and SSS triangles in geometry.
Questions to Develop Mathematical Thinking: <ul style="list-style-type: none">• Why can't you use the Law of Sines with a oblique triangle that is classified SAS or SSS?• Could SOH CAH TOA replace Law of Cosines? When and why?	Common Misconceptions: <ul style="list-style-type: none">• Law of Sines and Law of Cosines can be used on any oblique triangle.• SOH CAH TOA can always replace the Law of Sines and Law of Cosines.

Assessment (Formative and Summative):

<p>Formative Assessment</p> <ul style="list-style-type: none">• Bell work on <i>which triangles are ASA and AAS?</i> – previously taught material to reinforce these are triangle that fit the Law of Sines.• Class discussion on why the SAS and SSS triangle wouldn't work for the Law of Sines?• Worksheet A (Using the Law of Cosines to solve for the missing side length of an oblique triangle – SAS triangles only).• Student to student talk – Reflection on the worksheet and their answers.• Class discussion on the worksheet answers.• Class discussion on how Law of Cosines can be used in real life (student driven and teacher directed)<ul style="list-style-type: none">○ Naval Navigation example○ Forest fire example○ Deer hunter example• Exit ticket on the daily reflection• Forest Fire Lab (Real life/hands on application) <p>Summative Assessment</p> <p>Quiz on Law of Cosines that would include:</p> <ul style="list-style-type: none">• Oblique triangles SAS• Oblique triangles SSS• Real life application of both SAS and SSS

Materials:

<ul style="list-style-type: none">• Classroom overhead projector (Bell Work)• Law of Cosines Formula Page• Worksheet A• Exit tickets

WAMC Lesson Plan

Instruction Plan:

Introduction: The class will start with a bell work asking students to sort and identify oblique triangles that are ASA and AAS. There are 2 triangles that are SAS and SSS.

Explore: We will explore the meaning behind how the Law of Sines and Law of Cosines have their own place when faced with different types of oblique triangles.

When I observe students:

- My initial observation will be how successful students sort through and successfully select the oblique triangles (ASA and AAS) suitable for the Law of Sines (previously taught material).
- That observation will extend to making sure that all students understand how *SAS and SSS oblique triangles* do not work for the Law of Sines, thus introducing the Law of Cosines.
- After teaching and scaffolding the learning associated with the Law of Cosines (SAS oblique triangles only), I will observe students working independently on a worksheet specific to this skill set.
- I will then observe the student to student talking session as they compare their answers and ideas.
- I will then participate in the classroom discussion closing out this concept and observe students take the role as leader within this discussion and clarification/misconceptions associated with any errors that occurred. Specifically errors associated with mislabeling triangles and calculator errors (i.e. not enough parentheses or rounding within the middle of the problem).

Questions to Develop Mathematical Thinking as you observe:

- How did you decide what labels to use?
- How did you decide where to put alpha, beta and theta?
- Does it matter where you place your Cap A, B, or C?
- Do you see any real life application to this?

Answers:

- Answers will vary (some might say they used the same labels as the worksheet, others might not).
- I wrote Alpha opposite of A, Beta opposite of B and Theta opposite of C.
- No, but it does matter where Alpha, Beta and Theta are located based upon the placement of A, B and C.
- At this point the answers might vary from elaborate examples to students that just can't see any application.

Summarize:

Career Application(s):

- Construction
- Landscape design
- Natural Resource management
- Nautical Navigation
- Any profession that uses maps and triangulation

WAMC Lesson Plan

Leadership/21st Century Skills:

21st Century Interdisciplinary themes (Check those that apply to the above activity.)

- Global Awareness Financial/Economic/Business/Entrepreneurial Literacy Civic Literacy
 Health/Safety Literacy Environmental Literacy

21st Century Skills (Check those that students will demonstrate in the above activity.)

LEARNING AND INNOVATION

Creativity and Innovation

- Think Creatively
 Work Creatively with Others
 Implement Innovations

Critical Thinking and Problem Solving

- Reason Effectively
 Use Systems Thinking
 Make Judgments and Decisions
 Solve Problems

Communication and Collaboration

- Communicate Clearly
 Collaborate with Others

INFORMATION, MEDIA & TECHNOLOGY SKILLS

Information Literacy

- Access and Evaluate

Information

- Use and manage Information

Media Literacy

- Analyze Media
 Create Media Products

Information, Communications and Technology (ICT Literacy)

- Apply Technology Effectively

LIFE & CAREER SKILLS

Flexibility and Adaptability

- Adapt to Change
 Be Flexible

Initiative and Self-Direction

- Manage Goals and Time
 Work Independently

Social and Cross-Cultural

- Be Self-Directed Learners
 Interact Effectively with Others
 Work Effectively in Diverse Teams

Productivity and Accountability

- Manage Projects
 Produce Results

Leadership and Responsibility

- Guide and Lead Others
 Be Responsible to Others

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WAMC Lesson Plan

Lesson Plan Sequence:

- 1) Students will spend 5 minutes on the Bell Work “*which triangles are ASA and AAS?*”
 - a) This exercise is designed to help students identify triangles that are ASA and AAS to reinforce previously taught material (Law of Sines). This exercise introduces two triangles that are SAS and SSS. They are added to promote conversation surrounding how these two triangles would not work with the Law of Sines, thus allowing me to introduce the Law of Cosines.
- 2) Students will then be given the appropriate trigonometric formulas for the Law of Cosines (5 minutes):
 - a) $a^2 = b^2 + c^2 - 2bc\cos\alpha$
 - i. $a = \sqrt{b^2 + c^2 - 2bc\cos\alpha}$
 - b) $b^2 = a^2 + c^2 - 2ac\cos\beta$
 - i. $b = \sqrt{a^2 + c^2 - 2ac\cos\beta}$
 - c) $c^2 = a^2 + b^2 - 2ab\cos\theta$
 - i. $c = \sqrt{a^2 + b^2 - 2ab\cos\theta}$
- 3) The lesson will transition to how the Law of Cosines can be used to find missing side lengths (for SAS oblique triangles) – (15 minutes)
 - a) Several problems will be used to scaffold how a student can appropriately label a SAS triangle and set up the Law of Cosines with the supplied information in order to find the missing side length. These problems will be teacher led and questions/concerns will be answered as they arise (15 minutes).
 - i. The expectation is that students will need assistance with appropriately labeling the oblique triangles first and then identifying which formula best serves their needs based upon their own labels.
 - b) Once students appropriately identify which formula to use, they will then write the formula so that it can be more easily entered into a calculator for the final answer.
 - i. The expectation is that students will need assistance entering the formula into a calculator and identifying that the use of parentheses is crucial during this step.
- 4) After the problems are taught and students have the appropriate notes on how to solve SAS oblique triangle by use of the Law of Cosines, worksheet A will be passed out so students can independently work on these problems.
 - a) The teacher will formatively assess students individually at this time not to answer the questions for them, but simply guide them to how to appropriately label an oblique triangle and assist with the technology associated with deriving the answer (20 minutes).
- 5) Students will pair up (in their predetermined pairs) and discuss their findings/answers on Worksheet A. They will be given 5 minutes to discuss their answers and possibly debate differences.
- 6) The class will come together for a reflection and students will be allowed an opportunity to discuss their successes, failures, differences, who was right, who was wrong, and why...(5 minutes).
- 7) Class will end with an exit ticket on their reflection for their success today. They must answer the following question:
 - a) “What part of this process was easy for you? What part of this process was difficult for you? If you did not find any part of this process that was difficult, “How could the Law of Cosines be used as a real life application? Be specific in your answers. Turn in this exit ticket at the door as you leave the room.
- 8) The Lab for “Where is the Fire?” will be administered the following day (one complete class period).